



Smarter Balanced Assessment Consortium:

Stack of Cups Performance Task
Grade 8 Mathematics Practice Test
Scoring Guide
January 2017

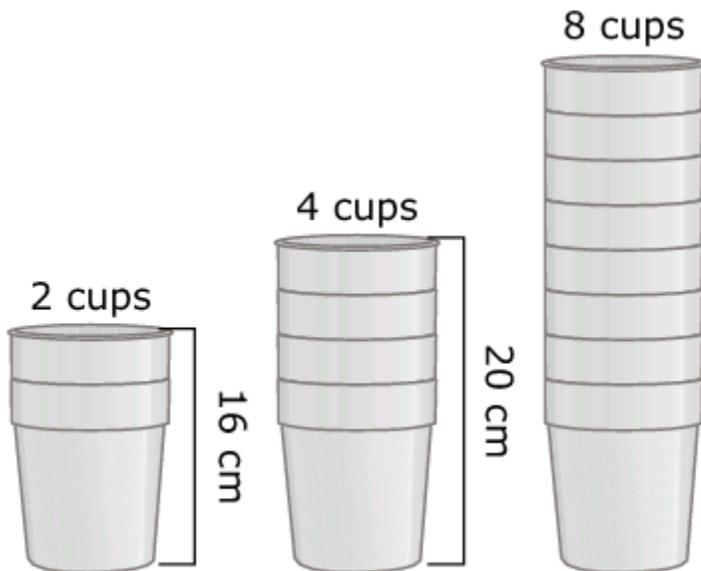


Grade 8 Mathematics

Stack of Cups Performance Task

Stacks of Cups

Your science classroom uses cups for many experiments. Your teacher ordered lots of cups from a catalog. The catalog is not very good. It has the following picture, but no other useful information.



Your teacher wants you to help her get organized for when the cups arrive next week. Using only the information shown in the picture, she asks you to figure out some other specific measurements.

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1

How tall, in cm, is the stack of 8 cups?

cm

2

How tall, in cm, is 1 cup? Explain how you determined the height of 1 cup.

3

Your teacher thinks that instead of having to figure out these stacks each time, it would be useful to understand the general relationship.

Write an equation expressing the relationship between the height of the stack and the number of cups in the stack.

Let h represent the height of the stack, in cm, and n the number of cups in the stack.

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4

The catalog is advertising a stack of these cups that is 95 cm tall. Lori says, "That must be a misprint because a stack of that height is not possible."

Do you agree or disagree with Lori? Explain your reasoning.

5

Your class wants to sell School Spirit Cups with your school logo on them. Your teacher wants you to design this new cup such that a stack of 10 cups will be 125 cm tall.

Describe key measurements of the School Spirit Cups and explain how they will meet the required specifications.

1

How tall, in cm, is the stack of 8 cups?

 cm

#1 Gridded response – 1 point

Item	Claim	Domain	Target	DOK	Content	MP	Key
#1	2	EE	2D	2	6.EE.B.7	2	28

Rubric:

1 point: Student responds with a value of 28, or equivalent.

0 points: All other responses

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Commentary:

This question is purposely easy in order to provide an accessible ramp into the overall task. The purpose of this question is to determine whether students can identify and extend the growth pattern that is occurring by stacking additional cups. The stack of 8 cups has been chosen purposefully because the pattern is not proportional and this shows stronger evidence of understanding of the linear relationship.

Rationale for Content:

This item requires students to interpret a real-world context and extend a pattern. Although students are not required to set up and solve an equation, the mathematical thinking involved to calculate the height of the 8 cups can be represented algebraically.

Rationale for Claim:

This item draws on the diagram provided to extend the growth pattern of the height of cups. Because students could solve this in a variety of ways (i.e., calculate the height of one cup and then find the height of 8 or calculate the height added per cup), this item qualifies as a Claim 2, Target D.

Claim 2, Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Rationale for DOK:

This is intended to be an entry-level question and maps onto DOK 2. From the *Depth of Thinking* chart:

ANALYZE: Extend a pattern

Because the students are not told which procedure to use and must explain their approach, this moves beyond DOK 1 and qualifies as DOK 2.

2

How tall, in cm, is 1 cup? Explain how you determined the height of 1 cup.

#2 Short text – 2 points

Item	Claim	Domain	Target	DOK	Content	MP	Key
#2	2	EE	2D	2	7.EE.B.3	2	1 cup is 14 cm tall with explanation

Rubric:

2 points: Student correctly calculates the height of one cup and provides a mathematically logical explanation as to how he/she calculated the height.

1 point: Student correctly calculates the height of one cup and provides an explanation that is not mathematically logical OR the student *only* calculates the height of a single cup.

0 points: All other responses

Commentary:

The purpose of this item is to continue to provide a ramp into the task that nonetheless provides evidence of problem solving (Claim 2) because the student is required to make sense of the situation, identify the relevant quantities, and relate these quantities to determine the solution. There are multiple solution paths and no single procedure nor path is provided; the students must develop their own approach. The numbers have been specifically chosen to not provide an arithmetic barrier of complexity, but to provide an accessible problem-solving opportunity.

Rationale for Content:

Because students are asked to identify the salient quantities and express them numerically to derive a solution, this item provides adequate evidence to support the designated content standard.

7.EE.B.3: Solve multi-step, real-life, and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Rationale for Claim:

To solve this problem, students must interpret and identify key values and measurements in the problem. They must do this in order to solve for the height of one single cup.

Claim 2, Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Rationale for DOK:

This item requires students to both select a procedure and perform it and retrieve information to solve a problem. For this reason, this item is a DOK 2, aligning to the categories of Apply and Analyze.

What follows are sample responses and scoring annotations for Item 2.

Sample Response 2a

B I U I_x
: = : = - = - =
✂ 📄 🗑 ↶ ↷
ABC
Ω

1 cup is 14 cm because if two cups is 16 and 4 cups is 20 then as cups stack 2 cm is added from each cup. To get to 20 from 16 its 4 cm. If only 2 cups are stacked, 16-2 is 14.

2

SCORE POINT

The student states the correct height for one cup and explains his/her method for determining the height.

Sample Response 2b

B I U I_x
: = : = - = - =
✂ 📄 🗑 ↶ ↷
ABC
Ω

1 cup is 14cm. To find this I compared the 2 cups and 4 cups then I subtracted 16 from 20 to get 4 so knowing the length of the top part of the cup, which was 2, I subtracted 2 from 16 to get 14.

2

SCORE POINT

The student states the correct height for the cup and provides an explanation as to how he/she calculated the height.

Sample Response 2c

B I U I_x
:≡ :≡ ≡ ≡
✂
📄
📁
↶
↷
ABC
Ω

14 cm. It looks like the top ridge is 2cm. So I subtracted 2 cm from 16cm and 14cm

SCORE POINT

1

The student states the correct height for the cup but does not provide a mathematically logical solution, and instead states, “It looks like the top ridge is 2cm.”

Sample Response 2d

B I U I_x
:≡ :≡ ≡ ≡
✂
📄
📁
↶
↷
ABC
Ω

I believe that 1 cup is about 14 cm tall I determined this by estimating that each extra cup in the stack adds on about 2 cm so each time I divided them and subtracted 2 cm, I got 14 cm tall for each cup.

SCORE POINT

1

The student states the correct height for the cup but provides an explanation that is not mathematically grounded. The student “determined this by estimating.”

Sample Response 2e

B I U I_x
:≡ :≡ ≡ ≡
✂
📄
📁
↶
↷
ABC
Ω

8 cm. If two are 16, divide by two to get the size of one.

SCORE POINT

0

Student incorrectly states the height and provides a mathematically incorrect approach for solving the problem.

3

Your teacher thinks that instead of having to figure out these stacks each time, it would be useful to understand the general relationship.

Write an equation expressing the relationship between the height of the stack and the number of cups in the stack.

Let h represent the height of the stack, in cm, and n the number of cups in the stack.

#3 Equation numeric – 1 point

Item	Claim	Domain	Target	DOK	Content	MP	Key
#3	4	EE	4F, 4E	2	7.EE.B.4	2	$h=2n+12$, or equivalent

Rubric:

1 point: Student writes an equation of the form $h = 2n + 12$ or an equivalent form.

0 points: All other responses

Commentary:

The purpose of this item is to gather evidence of the students' ability to generalize a linear growth pattern given values and examples.

Rationale for Content:

Because students are asked to write an algebraic equation that expresses the relationship between the number of cups and the height of the stack, this item provides evidence directly supporting "construct simple equations...to solve problems by reasoning about the quantities."

7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Rationale for Claim:

This item asks students to identify the relationship between the number of cups and the height of the stack, which is why this item qualifies as a Claim 4, Targets F and E.

4F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

Rationale for DOK:

Because students are asked to translate between a pictorial and algebraic representation, this item is DOK 2.

4

The catalog is advertising a stack of these cups that is 95 cm tall. Lori says, “That must be a misprint because a stack of that height is not possible.”

Do you agree or disagree with Lori? Explain your reasoning.

#4 Short text – 2 points

Item	Claim	Domain	Target	DOK	Content	MP	Key
#4	3	EE	3E	2	6.EE.B	3	See rubric

Rubric:

2 points: Student agrees with Lori and provides a valid mathematical explanation as to why a stack of cups could not reach 95 cm. For example, students could attend to the fact that all stacks are an even number of centimeters, or that when they plug in 95 for h to solve for n , it yields a non-whole number.

1 point: Student agrees with Lori, but provides a mathematical explanation that is incomplete.

0 points: Student disagrees with Lori, OR agrees with Lori, but does not offer any explanation for why.

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Note: There is a possible dependency on Item 3: If students did not write a correct equation in the previous item and this incorrect equation yields a whole number for n , students should be awarded full credit.

Commentary:

The purpose of this task is to assess the students' ability to communicate reasoning and their understanding of a quantity in context. In particular, the relationship between height of stack and number of cups yields a non-whole number of cups, corresponding to a height of 95 cm. In this context, that is not possible because we do not allow for parts of cups.

Rationale for Content:

Students are required to identify and make sense of quantities in a context, and reason about them. Namely, the relationship between height of stack and number of cups and the contextual restraint of the domain to whole numbers of cups.

6.EE.B: Reason about and solve one-variable equations and inequalities.

Rationale for Claim:

Because students are asked to directly agree or disagree with a conjecture and support their position, this item qualifies as Claim 3B.

Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjecture.

Rationale for DOK:

In this item, students need to "specify and explain relationships" as well as "make basic inferences or logical predictions." For these reasons, this item is a DOK 2.

5

Your class wants to sell School Spirit Cups with your school logo on them. Your teacher wants you to design this new cup such that a stack of 10 cups will be 125 cm tall.

Describe key measurements of the School Spirit Cups and explain how they will meet the required specifications.

#5 Short text – 2 points

Item	Claim	Domain	Target	DOK	Content	MP	Key
#5	4	EE	4A	3	7.EE.B.4	4	See sample responses

Rubric:

2 points: Student describes the key dimensions of the cup (height of cup, height of lip (if necessary)) and explains how 10 cups will reach a height of exactly 125 cm.

1 point: Student describes the key dimensions of the cup (height of cup, height of lip (if necessary)) that would satisfy the constraints, but does not explain how 10 cups will reach a height of exactly 125 cm.

0 points: Student does not describe key dimensions that would satisfy the constraints.

Commentary:

The purpose of this question is for students to design and describe a cup that when 10 of them are stacked, would reach a height of 125 cm. There are multiple solutions to this question, so it is imperative that students describe why their chosen dimensions meet the constraints.

Rationale for Content:

Although students do not necessarily have to set up an equation to solve this problem, students still need to reason about the quantities involved in the context. For that reason, this item addresses 7.EE.B.4.

7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Rationale for Claim:

This item satisfies Claim 4, Target A because the problem is not completely formulated. This falls under the construct of “design under constraints.”

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.

Rationale for DOK:

Because students need to “use concepts to solve non-routine problems” and “explain reasoning when more than one response is possible,” this item qualifies as DOK 3.

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What follows are sample responses and scoring annotations for Item 5.

Sample Response 5a

B I U I_x
:≡ :≡ ≡≡ ≡≡
✂
📄
📁
↶
↷
ABC
Ω

10 cm lips with 25 cm base is 35 cm tall each

The base of the cup is 25 cm, then 10 x 10cm for the lips will be 100cm for all of the lips. 100 +25 is 125 for the whole stack.

2

SCORE POINT

The student provides the key measurements of the cup (“lips” and “base”) and describes how these dimensions meet the required specifications.

Sample Response 5b

B I U I_x
:≡ :≡ ≡≡ ≡≡
✂
📄
📁
↶
↷
ABC
Ω

To make the cups how Karmin wants them to be, you would make the cups 12.5 cm tall and it would make a stack of 10 at 125cm tall.

1

SCORE POINT

The student partially explains the dimensions but fails to explain how they will meet the required specifications.

Sample Response 5c

B I U I_x
:≡ :≡ ≡≡ ≡≡
✂
📄
📁
↶
↷
ABC
Ω

-must be @ least 15 cm tall, individually
 -must be stirofoam
 -must be able to be divided by the height to get #

0

SCORE POINT

Student does not describe key dimensions for the height.

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The hand-scored items in this guide are 2-point short-text items. The general rubric that is used as a basis for scoring all 2-point short-text items is shown below. Although item-specific rubrics are also provided to scorers to facilitate the hand-scoring of short-text items, every response should be able to map back to this general rubric in a consistent and reliable manner.

Smarter Balanced Mathematics General Rubric for 2-Point Items

Score	Description
2	The student has demonstrated a full and complete understanding of all mathematical content and practices essential to this task. The student has addressed the task in a mathematically sound manner. The response contains evidence of the student’s competence in problem solving, reasoning, and/or modeling to the full extent that these processes apply to the specified task. The response may, however, contain minor flaws that do not detract from a demonstration of full understanding.
1	The student has demonstrated a partial understanding of the mathematical content and practices essential to this task. The student’s response contains some of the attributes of an appropriate response but lacks convincing evidence that the student fully comprehends the essential mathematical ideas addressed by this task. Such deficits include evidence of insufficient mathematical knowledge; errors in fundamental mathematical procedures; and other omissions or irregularities that bring into question the student’s competence in problem solving, reasoning, and/or modeling related to the specified task.
0	The student has demonstrated merely an acquaintance with the topic, or provided a completely incorrect or uninterpretable response. The student’s response may be associated with the task, but contains few attributes of an appropriate response. There are significant omissions or irregularities that indicate a lack of comprehension in regard to the mathematical content and practices essential to this task. No evidence is present that demonstrates the student’s competence in problem solving, reasoning, and/or modeling related to the specified task.