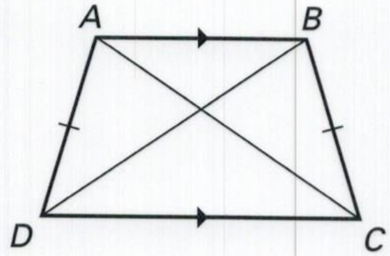
Answers for 8.6

For use with pages 554–558

8.6 Skill Practice

1. isosceles trapezoid
2. Prove its diagonals are perpendicular, each diagonal bisects a pair of opposite angles, or it has four congruent sides.
3. Rhombus, Square
4. ▱, Rectangle, Rhombus, Square
5. ▱, Rectangle, Rhombus, Square
6. Trapezoid
7. Rectangle, Square
8. Kite
9. Rhombus, Square, Kite
10. Rectangle, Square
11. ▱, Rectangle, Rhombus, Square
12. The fact that $\angle B$ and $\angle C$ are supplementary does not guarantee *ABCD* is a parallelogram. $\angle D$ and $\angle C$ are not supplementary, so $\overline{AD}$ is not parallel to $\overline{BC}$. So, *ABCD* is not a quadrilateral.
13. A
14. Rectangle; there are four right angles.
15. Trapezoid; there is one pair of parallel sides.
16. Kite; there are two pair of consecutive congruent sides.
17. 

$\overline{AC} \cong \overline{BD}$

isosceles trapezoid
18. No; it could be a kite.
19. No; $m\angle F = 109°$, which is not equal to $m\angle E$.
20. No; it could be a rectangle.
21. Kite; it has two pair of consecutive congruent sides.
22. Isosceles trapezoid; it has one pair of parallel sides and its diagonals are congruent.
23. Rectangle; opposite sides are parallel with four right angles.
24. Parallelogram; opposite sides are parallel but all four sides are not congruent and there is no right angle.

25. a. rhombus, square, kite

b. Parallelogram, rectangle, trapezoid; two consecutive pairs of sides are always congruent and one pair of opposite angles remain congruent.

26. $\overline{AB} \cong \overline{BC}$; all 4 sides are congruent and there are 4 right angles.

27. $m\angle B = 60°$ or $m\angle C = 120°$; then $\overline{AB} \parallel \overline{DC}$ and the base angles would be congruent.

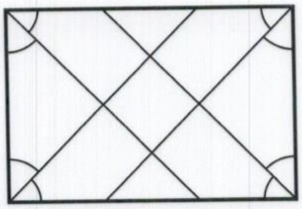
28. $\overline{DV} \cong \overline{BV}$; then the diagonals bisect each other.

29. No; if $m\angle JKL = m\angle KJM = 90°$, *JKLM* would be a rectangle.

30. Yes; *JKLM* has one pair of congruent base angles and only one pair of parallel sides.

31. Yes; *JKLM* has one pair of non-congruent parallel sides with congruent diagonals.

32.



Square; when the rectangle's angles are bisected, the resulting angle measures are 45°. The triangles created all have angle measures 45°-45°-90° and are similar. So the quadrilateral has four right angles since each is one of a pair of vertical angles where the other angle is a right angle. Pairs of angle bisectors are parallel since they are prependicular to the same line (one of the other angle bisectors). Therefore, the quadrilateral is a parallelogram, making its opposite sides congruent. Consecutive sides of the quadrilateral can be shown congruent using congruent triangles and the Subtraction Property of Equality. Therefore, the quadrilateral has four congruent sides and four right angles, which makes it a square.

8.6 Problem Solving

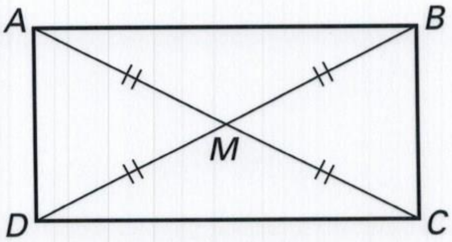
33. trapezoid **34.** kite

35. parallelogram

36. a. trapezoid **b.** rectangle

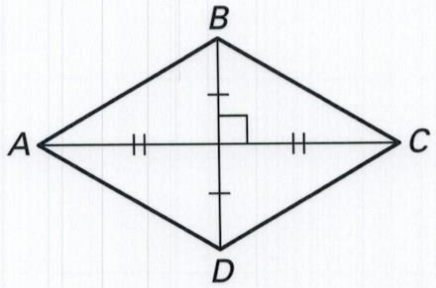
37. Consecutive interior angles are supplementary making each interior angle 90°.

38. a.



$\overline{AC}$ and $\overline{BD}$ are diagonals of quadrilateral *ABCD*. Since they are congruent, *ABCD* is either a rectangle or a square; since they are not perpendicular, *ABCD* must be a rectangle.

b.



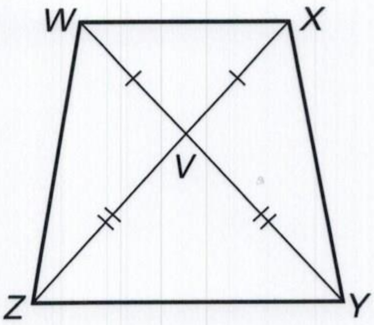
Since $\overline{AC}$ and $\overline{BD}$ are perpendicular, quadrilateral *ABCD* is a rhombus, square, or kite; since they are not congruent, *ABCD* is a rhombus or a kite. However, the diagonals of a kite do not bisect each other, so *ABCD* must be a rhombus.

39. a. Using the definition of a regular hexagon, $\overline{UV} \cong \overline{VQ} \cong \overline{RS} \cong \overline{ST}$ and $\angle V \cong \angle S$. So $\triangle QVU$ and $\triangle RST$ are isosceles. Using the SAS Congruence Postulate, $\triangle QVU \cong \triangle RST$.

b. Using the definition of a regular hexagon, $\overline{QR} \cong \overline{UT}$. Using corresponding parts of congruent triangles are congruent, $\overline{QU} \cong \overline{RT}$.

c. Since $\angle Q \cong \angle R \cong \angle T \cong \angle U$ and $\angle VUQ \cong \angle VQU \cong \angle STR \cong \angle SRT$, you know that $\angle UQR \cong \angle QRT \cong \angle RTU \cong \angle TUQ$ by the Angle Addition Postualte; 90°.

d. Rectangle; there are 4 right angles and opposite sides are congruent.

40. 

isosceles trapezoid; show $\overline{WX} \parallel \overline{ZY}$ by showing $\triangle WVX \sim \triangle YVZ$ which leads to $\angle XWV \cong \angle ZYV$. Now show $\angle ZWX \cong \angle YXW$ using $\triangle ZVW \cong \triangle YVX$ and $\angle XWV \cong \angle WXV$.

41. Square; Given: $PQRS$ is a square with E, F, G, and H midpoints of the square. Using the definition of a square and the definition of midpoint, $\overline{FQ} \cong \overline{QG} \cong \overline{GR} \cong \overline{RH} \cong \overline{HS} \cong \overline{SE} \cong \overline{PE} \cong \overline{PF}$. Using the definition of square, $\angle P \cong \angle Q \cong \angle R \cong \angle S$. Using the SAS Congruence Theorem, $\triangle EPF \cong \triangle FQG \cong \triangle GRH \cong \triangle HSE$. Using corresponding parts of congruent triangles are congruent, $\overline{EF} \cong \overline{FG} \cong \overline{GH} \cong \overline{HE}$. Since the base angles of all four triangles measure 45°, $m\angle EFG = m\angle FGH = m\angle GHE = m\angle HEF$. By definition, $PQRS$ is a square.

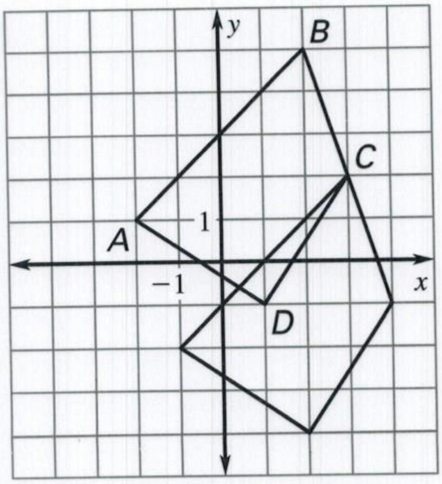
42. Rhombus; Given: Three-dimensional figure and $\overline{JK} \cong \overline{LM}$; E, F, G and H are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$, respectively. Using the definition of midsegment $\overline{FG}$ and $\overline{EH}$ are parallel to $\overline{LM}$ and half of its length. This makes $\overline{FG} \parallel \overline{EH}$ and $\overline{FG} \cong \overline{EH}$. Using the definition of midsegment, $\overline{GH}$ and $\overline{FE}$ are parallel to $\overline{JK}$ and half of its length. This makes $\overline{GH} \parallel \overline{FE}$ and $\overline{GH} \cong \overline{FE}$. Since $\overline{JK} \cong \overline{LM}$, you know that $\overline{FG} \cong \overline{EH} \cong \overline{GH} \cong \overline{FE}$ by the Transitive Property of Congruence. By definition, $EFGH$ is a rhombus.

8.6 Mixed Review

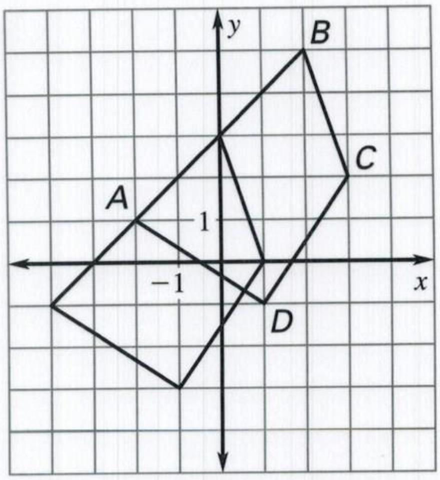
43. 4, 7; since $\triangle ABD$ is equilateral, all three sides are congruent and are 7 units long. Since $\triangle ADC$ is isosceles, $AD = 7 = CD$, thus $x + 3 = 7$ or $x = 4$.

44. 120°, 30°, 30°; since $\triangle ABD$ is equiangular, $m\angle BDA = 60°$ making $m\angle ADC = 120°$ by definition of a linear pair. Since $m\angle DAC = m\angle DCA$ and $m\angle ADC = 120°$, the Triangle Sum Theorem indicates $m\angle DAC = m\angle DCA = 30°$.

45.



46.



47. 12

48. 5

49. 4

50. 8

8.4–8.6 Mixed Review of Problem Solving

1. a. *ABCD* has one pair of parallel sides.

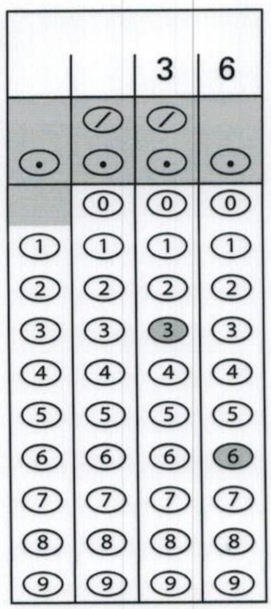
b. Yes; the base angles are congruent.

2. Yes; since the four triangles are congruent right triangles you know that $\overline{JK} \cong \overline{KL} \cong \overline{LM} \cong \overline{MJ}$ with $\angle J \cong \angle K \cong \angle L \cong \angle M$ each measuring 90°.

3. a. 66.5°; in a kite one pair of opposite angles have the same measure, therefore $m\angle QTS = m\angle QRS$. Now $2m\angle QTS =$ $360° - (120° + 105°)$ making $m\angle QTS = 66.5°$.

b. $7\sqrt{2} \approx 10$ ft, $\sqrt{65} \approx 8$ ft, $\sqrt{65} \approx 8$ft, $7\sqrt{2} \approx 10$ ft; since $TR = TP + RP = 14$ and $TP = QP + RP$, this makes each of these segments 7 feet long and $PS = 4$ feet. Use the Pythagorean Theorem to find the lengths of the sides of the kite.

4. 36 in.;



5. 25; since the rhombuses are similar, corresponding parts are proportional. Solve $\frac{32}{40} = \frac{20}{x}$ to find WX.

6. a. Rectangle, square, isosceles trapezoid; the diagonals of these three quadrilaterals are congruent.

b. For a rectangle you need to know that opposite sides are congruent. For a square you need to know that opposite sides are congruent and that consecutive sides are congruent. For an isosceles trapezoid you need to know that only one pair of opposite sides are parallel.

7. a. $H(2, 6)$; if H is at $(2, 6)$, $HG = GF = FE = EH$, $\overline{HG} \parallel \overline{FE}$, and $\overline{EH} \parallel \overline{FG}$. If H is at $(-2, 2)$, both pairs of opposite sides are parallel and congruent, but the quadrilateral is $EGFH$, not $EFGH$.

b. *Sample answer:* The coordinates of H could be $(2, 10)$ or $(2, 14)$. Both points allow $EFGH$ to meet the definition of a kite. The points lie on the line with equation $x = 2$ excluding $(2, 6)$ and $(2, 2)$.