Answers for 8.4

For use with pages 537–540

8.4 Skill Practice

1. square

2. No; *WXYZ* is not known to be a parallelogram.

3–8. Check students' diagrams.

3. Sometimes; *JKLM* would need to be a square.

4. Always; in a rhombus opposite pairs of angles are always congruent.

5. Always; in a rhombus all four sides are congruent.

6. Always; in a rhombus all four sides are congruent.

7. Sometimes; diagonals are congruent if the rhombus is a square.

8. Always; diagonals of a rhombus bisect the interior angles.

9–14. Check students' diagrams.

9. Always; in a rectangle all interior angles measure 90°.

10. Always; in a rectangle opposite pairs of sides are congruent.

11. Sometimes; adjacent sides are congruent if the rectangle is a square.

12. Always; diagonals of a rectangle are congruent.

13. Sometimes; diagonals are perpendicular if the rectangle is a square.

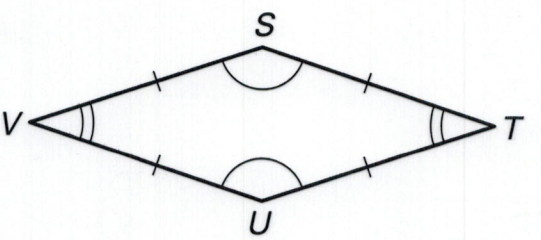
14. Sometimes; interior angles are bisected if the rectangle is a square.

15. Square; the quadrilateral has four congruent sides and angles.

16. Rectangle; both pairs of opposite sides are congruent and all of the angles are right angles.

17. Rhombus. *Sample answer:* The fourth angle measure is 40°, meaning that both pairs of opposite sides are parallel. So the figure is a parallelogram with two consecutive sides congruent. But this is only possible if the remaining two sides are also congruent, so the quadrilateral is a rhombus.

18.



STUV is a parallelogram with four congruent sides, opposite pairs of sides are parallel, opposite pairs of angles are congruent, and perpendicular diagonals bisect each other and bisect opposite angles.

19. rectangle, square

20. square

21. rhombus, square

22. parallelogram, rectangle, rhombus, square

23. parallelogram, rectangle, rhombus, square

24. rhombus, square

25. $7x - 4$ is not necessarily equal to $3x + 14$; $(7x - 4) + (3x + 14) = 90$, $10x + 10 = 90$, $10x = 80$, $x = 8$.

26. Rhombus; *ABCD* is a quadrilateral with four congruent sides; 76, 4.

27. Rectangle; *JKLM* is a quadrilateral with four right angles; 10, 15.

28. Square; *PQRS* is a quadrilateral with four right angles and perpendicular diagonals; 9, 5.

29. Parallelogram; *EFGH* is a quadrilateral with opposite pairs of sides congruent; 13, 2.

30. 20 in.; four 3-4-5 right triangles are created by the diagonals.

31. C

32. 53°

33. 90°

34. 74°

35. 16

36. about 6.0

37. about 12.0

38. 56°

39. 112°

40. 5

41. 5

42. about 8.3

43. about 5.6

44. 90°

45. 45°

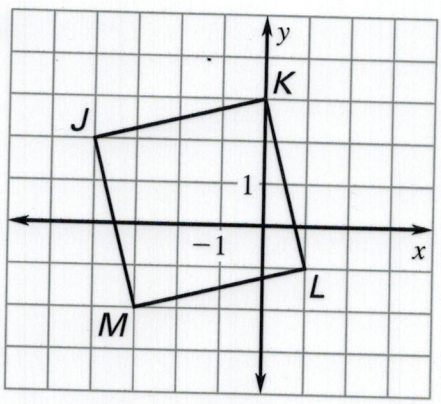
46. 45°

47. 1

48. 2

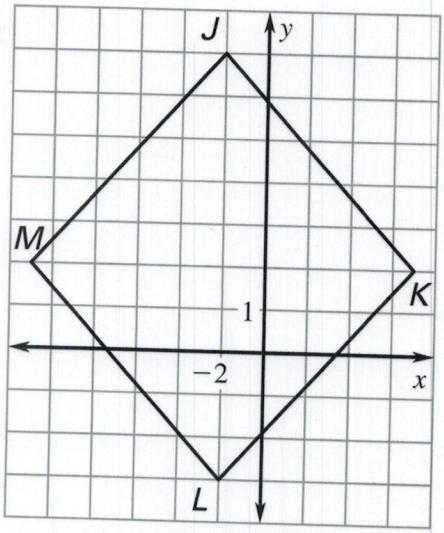
49. $\sqrt{2}$

50.



square; opposite sides are parallel, adjacent sides are perpendicular, and all four sides are congruent; $4\sqrt{17}$.

51.



rhombus; four congruent sides and opposite sides are parallel; $4\sqrt{106}$.

52. No; yes; two rhombuses do not have to have the same angle measures; all squares have the same angle measures, and the ratios of the lengths of their sides will be equal.

53. $AB = BC = CD = DA = \sqrt{89}$; $m\angle ABC = m\angle ADC = 64°$, $m\angle DAB = m\angle BCD = 116°$. The diagonals of a rhombus bisect each other and intersect at a right angle to form four congruent right triangles. The Pythagorean Theorem is used to find the side lengths and the tangent ratio is used to compute the measures of the angles of the rhombus.

8.4 Problem Solving

54. a. Rhombus, rectangle; $HBDF$ is a rhombus since all four sides are congruent and $ACEG$ is a rectangle since all four angles are right angles.

b. They are congruent; they are congruent; the diagonals of a rectangle are congruent and they bisect each other.

55. Measure the diagonals. If they are the same, it is a square.

56. Since the diagonals of a parallelogram bisect each other $AX = XC$. $BX = BX$ by the Reflexive Property, and it is given that $AB = BC$. $\triangle AXB \cong \triangle CXB$ by the SSS Congruence Theorem. Because corresponding parts of congruent triangles are congruent, $\angle AXB \cong \angle CXB$ and since they are a linear pair, they both must be 90°, which means $\overline{AC} \perp \overline{BD}$.

57. If a quadrilateral is a rhombus, then it has four congruent sides; if a quadrilateral has four congruent sides, then it is a rhombus; the conditional statement is true since a quadrilateral is a parallelogram and a rhombus is a parallelogram with four congruent sides; the converse is true since a quadrilateral with four congruent sides is also a parallelogram with four congruent sides making it a rhombus.

58. If a quadrilateral is a rectangle, then it has four right angles; if a quadrilateral has four right angles, then it is a rectangle; the conditional statement is true since the definition of a rectangle includes four right angles; the converse is true since four right angles will lead to parallel sides.

59. If a quadrilateral is a square, then it is a rhombus and a rectangle; if a quadrilateral is a rhombus and a rectangle, then it is a square; the conditional statement is true since a square is a parallelogram with four right angles and four congruent sides; the converse is true since a rhombus has four congruent sides and the rectangle has four right angles and thus a square follows.

60. Since $PQRS$ is a parallelogram the diagonals bisect each other, making $\overline{PT} \cong \overline{RT}$. This along with what is given implies $\triangle PQT \cong \triangle RQT \cong \triangle RST \cong \triangle PST$ using the AAS Congruence Theorem. It follows that $\overline{QR} \cong \overline{SR} \cong \overline{SP} \cong \overline{QP}$ using corresponding parts of congruent triangles are congruent. Since all four sides of a parallelogram are congruent, it is a rhombus.

61. Since $WXYZ$ is a rhombus the diagonals are perpendicular, making $\triangle WVX$, $\triangle WVZ$, $\triangle YVX$, and $\triangle YVZ$ right triangles. Since $WXYZ$ is a rhombus, $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$. Using Theorem 8.6, $\overline{WV} \cong \overline{YV}$ and $\overline{ZV} \cong \overline{XV}$. Now $\triangle WVX \cong \triangle WVZ \cong \triangle YVX \cong \triangle YVZ$. Using corresponding parts of congruent triangles are congruent, you now know $\angle WVZ \cong \angle WVX$ and $YVZ \cong \angle YVX$ which shows $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$. Similarly $\angle VZW \cong \angle VZY$ and $\angle VXW \cong \angle VXY$. This shows $\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$.

62. a. $\angle ABD \cong \angle CDB$ using the Alternate Interior Angles Congruence Theorem; $\angle ADB \cong \angle ABD$; $\overline{AB} \cong \overline{AD}$; $\angle ADB \cong \angle ABD$ using the Transitive Property of Angle Congruence and $\overline{AB} \cong \overline{AD}$ using the Converse of Base Angles Theorem.

b. Rhombus; if $\overline{AD} \parallel \overline{BC}$, then the quadrilateral is a parallelogram by definition. Using the fact that opposite sides of a parallelogram are congruent along with the fact that $\overline{AB} \cong \overline{AD}$, as shown in part (a), means all four sides of the parallelogram are congruent. Therefore, $ABCD$ is a rhombus by definition.

63. *Sample answer:* Let rectangle $ABCD$ have vertices $(0, 0)$, $(a, 0)$, (a, b), and $(0, b)$ respectively. The diagonal $\overline{AC}$ has a length of $\sqrt{a^2 + b^2}$ and diagonal $\overline{BD}$ has a length of $\sqrt{a^2 + b^2}$. So, $AC = BD = \sqrt{a^2 + b^2}$.

64. *Sample answer:* Let parallelogram $DFGH$ have vertices $D\left(a, \sqrt{b^2 - a^2}\right)$, $F(b, 0)$, $G\left(-a, -\sqrt{b^2 - a^2}\right)$, and $H(-b, 0)$, respectively. The slope of $\overline{HG}$ and $\overline{DF}$ is $\frac{\sqrt{b^2 - a^2}}{a - b}$ and the slope of $\overline{HD}$ and $\overline{GF}$ is $\frac{\sqrt{b^2 - a^2}}{a + b}$. The product of the slopes of $\overline{HG}$ and $\overline{GF}$, $\overline{GF}$ and $\overline{DF}$, $\overline{DF}$ and $\overline{HD}$, and $\overline{HD}$ and $\overline{HG}$ are -1, making each pair of consecutive segments perpendicular and each angle is a right angle. Therefore, parallelogram $DFGH$ is a rectangle.

8.4 Mixed Review

65. 27.1 cm

66. $\frac{24}{25}$ or 0.96, $\frac{7}{25}$ or 0.28

67. $\frac{7}{25}$ or 0.28, $\frac{24}{25}$ or 0.96

68. 99 **69.** 120 **70.** 45

Quiz
1) 4
2) 16
3) 24
4) Square (both rhom & rect)
5) rhombus Thm 8.12 (diag bisects opp. angles)
6) rectangle Rect. corollary (4 rt ∠s)