Answers for 8.3

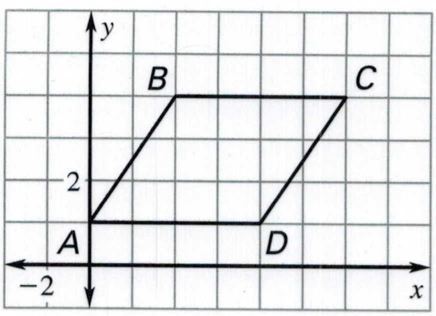
For use with pages 526–532

8.3 Skill Practice

1. The definition of a parallelogram is that it is a quadrilateral with opposite pairs of parallel sides. Since $\overline{AB}$, $\overline{CD}$, and $\overline{AD}$, $\overline{BC}$ are opposite pairs of parallel sides, quadrilateral $ABCD$ is a parallelogram.
2. Yes; both pairs of opposite sides are congruent.
3. The congruent sides must be opposite one another.
4. Theorem 8.8
5. Theorem 8.7
6. Theorem 8.10
7. Since both pairs of opposite sides of $JKLM$ always remain congruent, $JKLM$ is always a parallelogram and $\overline{JK}$ remains parallel to $\overline{ML}$.

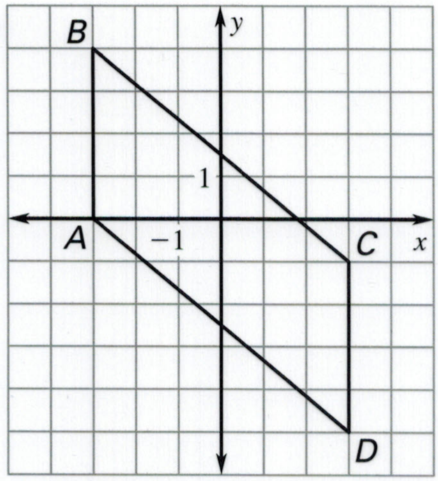
8. 4 **9.** 8 **10.** $\frac{2}{3}$

11.



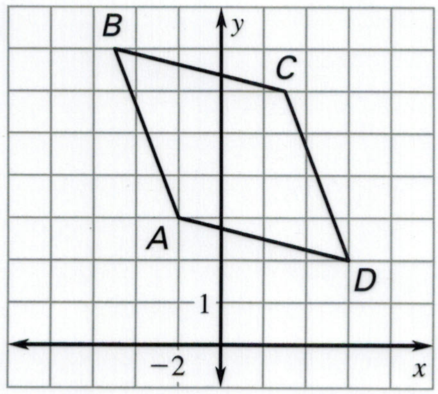
Sample answer: $AB = CD = 5$ and $BC = DA = 8$

12.



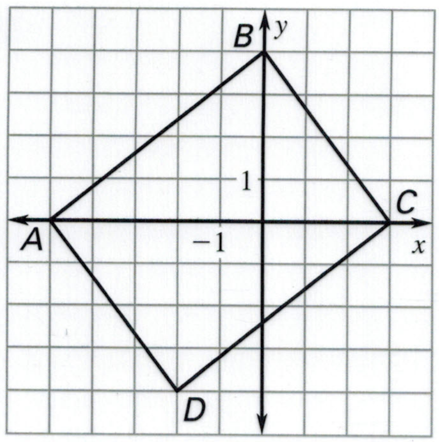
Sample answer: $AB = CD = 4$ and $BC = DA = \sqrt{61}$

13.



Sample answer: $AB = CD = 5$ and $BC = DA = \sqrt{65}$

14.



Sample answer: $AB = CD = \sqrt{41}$ and $BC = DA = 5$

15. *Sample answer:* Show $\triangle ADB \cong \triangle CBD$ using the SAS Congruence Postulate. This makes $\overline{AD} \cong \overline{CB}$ and $\overline{BA} \cong \overline{CD}$ using corresponding parts of congruent triangles are congruent.

16. *Sample answer:* Show $\triangle ADB \cong \triangle CBD$ using the ASA Congruence Theorem. This makes $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$ using corresponding parts of congruent triangles are congruent.

17. *Sample answer:* Show $\overline{AB} \parallel \overline{DC}$ by the Alternate Interior Angles Converse, and show $\overline{AD} \parallel \overline{BC}$ by the Corresponding Angles Converse.

18. A

19. 114

20. 45

21. 50

22. $PQRS$ is a parallelogram if and only if $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

23. $PQRS$ is a parallelogram if and only if $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

24. Joining the endpoints of the two line segments that bisect one another forms a quadrilateral. Using Theorem 8.10 you know that it is a parallelogram.

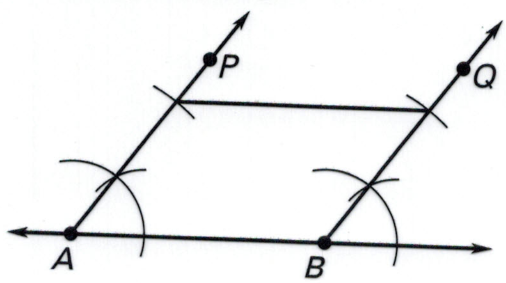
25. $(-3, 2)$; since $\overline{DA}$ must be parallel and congruent to $\overline{BC}$, use the slope and length of $\overline{BC}$ to find point D by starting at point A.

26. $(3, 1)$; since $\overline{AB}$ must be parallel and congruent to $\overline{CD}$, use the slope and length of $\overline{AB}$ to find point D by starting at point C.

27. $(-5, -3)$; since $\overline{DA}$ must be parallel and congruent to $\overline{BC}$, use the slope and length of $\overline{BC}$ to find point D by starting at point A.

28. $(7, -2)$; since $\overline{AB}$ must be parallel and congruent to $\overline{CD}$, use the slope and length of $\overline{AB}$ to find point D starting at point C.

29. *Sample answer:* Draw a line passing through points A and B. At points A and B, construct $\overrightarrow{AP}$ and $\overrightarrow{BQ}$ such that the angle each ray makes with the line is the same. Mark off congruent segments starting at points A and B along $\overrightarrow{AP}$ and $\overrightarrow{BQ}$, respectively. Draw the line segment joining these two endpoints.



30. 8; since $ABCD$ is a parallelogram you know that $\overline{AD} \cong \overline{CB}$ and $\overline{AD} \parallel \overline{CB}$. $\angle ADE \cong \angle CBF$ using the Alternate Interior Angles Congruence Theorem. It was given that $\overline{BF} \cong \overline{DE}$, therefore $\triangle ADE \cong \triangle CBF$ by the SAS Congruence Theorem. Using corresponding parts of congruent triangles are congruent, you know that $CF = EA = 8$.

8.3 Problem Solving

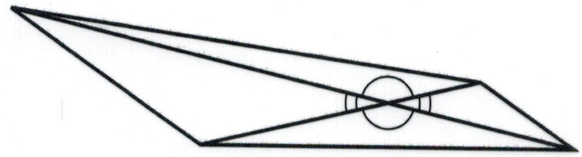
31. a. $EFJK$, $FGHJ$, $EGHK$; in each case opposite pairs of sides are congruent.

b. Since $EGHK$ is a parallelogram, opposite sides are parallel.

32. $AEFD$ and $EBCF$ are parallelograms by Theorem 8.8 so $\overline{AD}$ and $\overline{BC}$ both remain parallel to $\overline{EF}$; $\overline{AE}$ and $\overline{DF}$, $\overline{BE}$ and $\overline{CF}$.

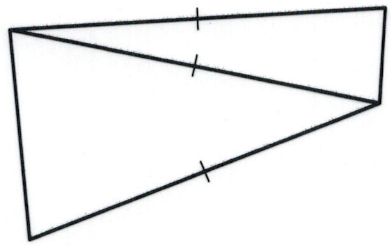
33. 1st column: Alternate Interior Angles Congruence Theorem; Reflexive Property of Segment Congruence; Given
2nd column: SAS; Corr. Parts of $\cong$ $\triangle$s are $\cong$; Theorem 8.7

34.



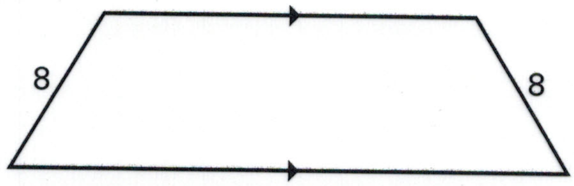
The point of intersection of the diagonals is not necessarily their midpoint.

35.



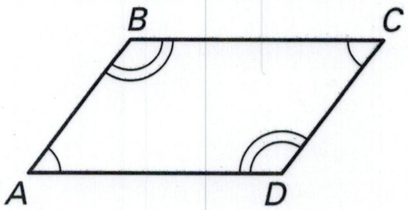
The opposite sides that are not marked in the given diagram are not necessarily the same length.

36.



The sides of length 8 are not necessarily parallel.

37. In a quadrilateral, if consecutive angles are supplementary, then the quadrilateral is a parallelogram. In $ABCD$ you are given $\angle A$ and $\angle B$ are supplementary, and $\angle C$ and $\angle B$ are supplementary, which gives you $m\angle A = m\angle C$. Also $\angle B$ and $\angle C$ are supplementary, and $\angle C$ and $\angle D$ are supplementary which gives you $m\angle B = m\angle D$. So $ABCD$ is a parallelogram by Theorem 8.8.

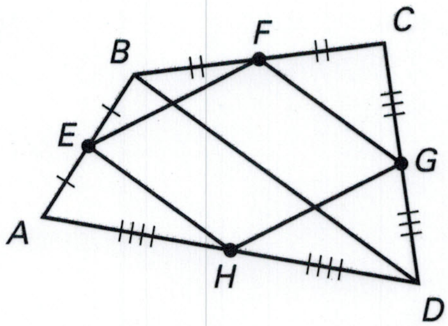


38. It is given that $\angle A \cong \angle C$ and $\angle B \cong \angle D$. Let $m\angle A = m\angle C = x$ and $m\angle B = m\angle D = y$. Since $ABCD$ is a quadrilateral, you know that $2x + 2y = 360$ using the Polygon Interior Angles Theorem, so $x + y = 180$. Using the definition of supplementary angles, $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, and $\angle D$ and $\angle A$ are supplementary. Using Theorem 8.5, $ABCD$ is a parallelogram.

39. It is given that $\overline{KP} \cong \overline{MP}$ and $\overline{JP} \cong \overline{LP}$ by definition of segment bisector. $\angle KPL \cong \angle MPJ$ and $\angle KPJ \cong \angle MPL$ since they are vertical angles. $\triangle KPL \cong \triangle MPJ$ and $\triangle KPJ \cong \triangle MPL$ by the SAS Congruence Postulate. Using corresponding parts of congruent triangles are congruent, $\overline{KJ} \cong \overline{ML}$ and $\overline{JM} \cong \overline{LK}$. Using Theorem 8.7, $JKLM$ is a parallelogram.

40. It is given that $DEBF$ is a parallelogram and $AE = CF$. Since $DEBF$ is a parallelogram, you know that $FD = EB$, $\angle BFD \cong \angle DEB$, and $ED = FB$. $AE + EB = CF + FD$ which implies that $AB = CD$, which implies that $\overline{AB} \cong \overline{CD}$. $\angle BFC$ and $\angle BFD$, and $\angle DEB$ and $\angle DEA$ form linear pairs, thus making them supplementary. Using the Congruent Supplements Theorem, $\angle BFC \cong \angle DEA$ making $\triangle AED \cong \triangle CFB$ using SAS. Using corresponding parts of congruent triangles are congruent, $\overline{AD} \cong \overline{CB}$. Theorem 8.7 tells you that $ABCD$ is a parallelogram.

41.

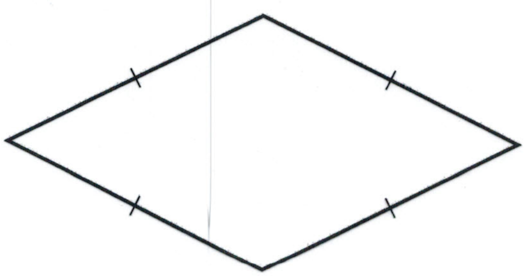


Sample answer: Consider the diagram. $\overline{FG}$ is the midsegment of $\triangle CBD$ and therefore is parallel to $\overline{BD}$ and half of its length. $\overline{EH}$ is the midsegment of $\triangle ABD$ and therefore is parallel to $\overline{BD}$ and half of its length. This makes $\overline{EH}$ and $\overline{FG}$ both parallel and congruent. Using Theorem 8.9, $EFGH$ is a parallelogram.

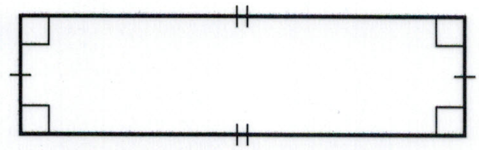
42. $\overline{FJ}$ is the midsegment of $\triangle AED$ and therefore is parallel to $\overline{AD}$ and half of its length. $\overline{GH}$ is the midsegment of $\triangle BEC$ and therefore is parallel to $\overline{BC}$ and half of its length. Together this gives you $\overline{FJ} \cong \overline{GH}$ and $\overline{FJ} \parallel \overline{GH}$. Using Theorem 8.9, $FGHJ$ is a parallelogram.

8.3 Mixed Review

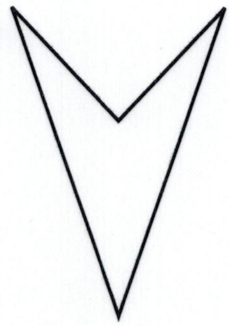
43.



44.



45.



46. 106.25 cm^2 **47.** $4\sqrt{3}$, 8

8.1–8.3 Mixed Review of Problem Solving

1. **a.** 5; pentagon

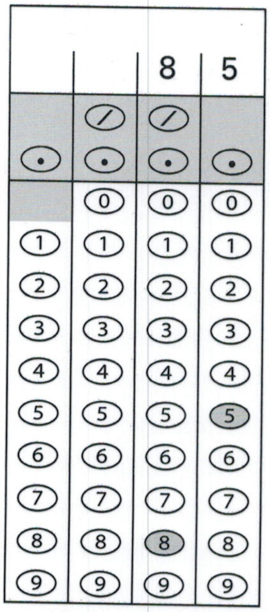
b. 540°

c. 360°

2. The sum of the interior angles of $ABCDE$ is 540°. To find $m\angle A$ and $m\angle C$, subtract 270° from 540° and divide by 2.

3. $x = 4$, $y = 4$; the diagonals of a parallelogram bisect each other. Set $12x + 1 = 49$ and $8y + 4 = 36$ and solve for x and y.

4. 85;



5. In a parallelogram consecutive interior angles are supplementary. Solve $x + 3x - 12 = 180$ for x. Use the value of x to find the degree measure of the two consecutive angles. In a parallelogram the angle opposite each of these known angles has the same measure.

6. a. *EFGH* is a parallelogram by Theorem 8.9. As *EFGH* changes the shape, $\angle E$ and $\angle G$ remain congruent, and $\angle H$ and $\angle F$ remain congruent, keeping $\overline{FG} \parallel \overline{EH}$.

b. $m\angle E$ and $m\angle G$ decrease from 55° to 50°. $m\angle H$ and $m\angle F$ change from 125° to 130°. Since *EFGH* is always a parallelogram, $m\angle F$ and $m\angle H$ are always the same and $m\angle E$ and $m\angle G$ are always the same. Since $m\angle E$ is always supplementary to the $m\angle F$, their sum must be 180°.

7. a. Since the slope of $\overline{MN}$ and $\overline{PQ}$ is $\frac{3}{11}$ and the slope of $\overline{NP}$ and $\overline{QM}$ is $-\frac{5}{4}$, then it is a parallelogram by definition.

b. $MN = PQ = \sqrt{130}$ making $\overline{MN} \cong \overline{PQ}$ and $NP = QM = \sqrt{41}$ making $\overline{NP} \cong \overline{QM}$. Using the Theorem 8.3, *MNPQ* is a parallelogram.

8. $\overline{BX} \parallel \overline{DY}$ using the Lines Perpendicular to a Transversal Theorem. Since $\overline{BX} \perp \overline{AC}$ and $\overline{DY} \perp \overline{AC}$, then $\angle BXA$ and $\angle DYC$ are right angles making $\angle BXA \cong \angle DYC$. Using the Alternate Interior Angles Congruence Theorem, $\angle BAX \cong \angle DCY$. $\overline{AB} \cong \overline{CD}$ since opposite sides of a parallelogram are congruent. $\triangle BXA \cong \triangle DYC$ by the AAS Congruence Theorem, making $\overline{BX} \cong \overline{DY}$ using corresponding parts of congruent triangles are congruent. Use Theorem 8.9 to show *XBYD* is a parallelogram.