Answers for 5.4

For use with pages 322–325

5.4 Skill Practice

1. circumcenter: when it is an acute triangle, when it is a right triangle, when it is an obtuse triangle; incenter: always, never, never; centroid: always, never, never; orthocenter: when it is an acute triangle, when it is a right triangle, when it is an obtuse triangle.

2. Both are perpendicular to a side of the triangle, although the altitude contains the vertex opposite the side while a perpendicular bisector bisects the side but does not necessarily contain the opposite vertex; both bisect one side of a triangle, although the perpendicular bisector does not necessarily contain the opposite vertex while the median is not necessarily perpendicular to the side but does contain the opposite vertex.

3. 12 **4.** 9 **5.** 10

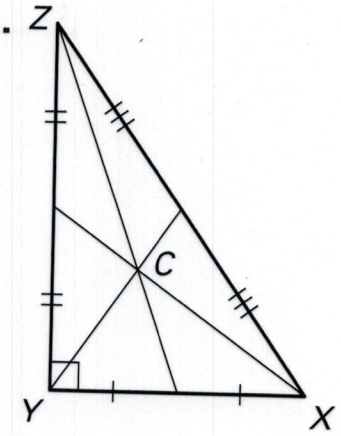
6. 5 **7.** D

8. a. (8, 1); (5, 1)

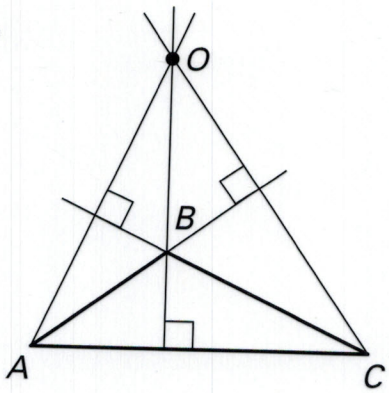
b. (5, −1); $SQ = 4$ and $SR = 6$, therefore $SQ = \frac{2}{3}SR$.

9. (3, 2) **10.** (3, 4)

11.



12.



13. no; no; yes

14. yes; yes; yes

15. no; yes; no

16. T is the orthocenter, but the centroid is needed to reach the conclusion.

17. altitude

18. angle bisector

19. median

20. perpendicular bisector, angle bisector, median, altitude

21. perpendicular bisector, angle bisector, median, altitude

22. perpendicular bisector, angle bisector, median, altitude

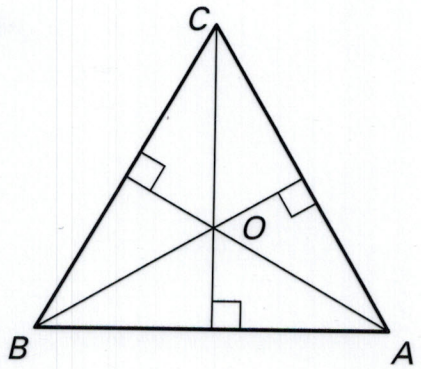
23. 6, 22°; $\triangle ABD \cong \triangle CBD$ by HL, use corr. parts of $\cong \triangle$s are $\cong$.

24. 90°, 22°; $\triangle ABD \cong \triangle CBD$ by SSS, use definition of a linear pair and corr. parts of $\cong \triangle$s are $\cong$.

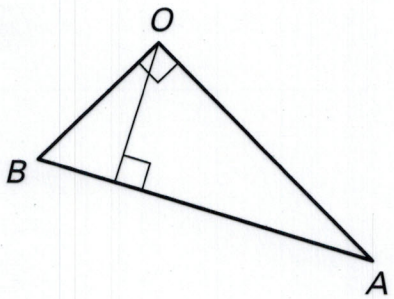
25. 3 **26.** 2 **27.** $\frac{3}{2}$

28. If the base angles of the isosceles triangle are placed at $(-a, 0)$ and $(0, a)$, the vertex angle will be on the y-axis.

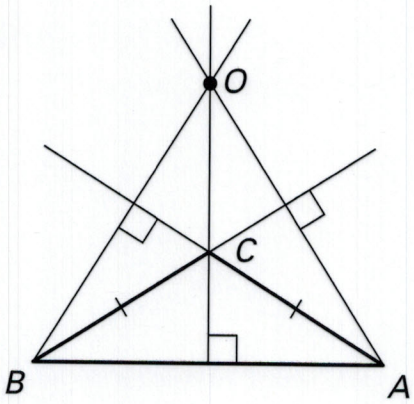
29.



30.



31.



32. *Sample answer:* The midpoint of $\overline{FG}$ is $L(3, 7)$, so the equation of the median from $H(6, 1)$ to $L(3, 7)$ is $y = -2x + 13$. $P(4, 5)$ lies on this median. The midpoint of $\overline{GH}$ is $J(5, 5)$, so the equation of the median from $F(2, 5)$ to $J(5, 5)$ is $y = 5$. $P(4, 5)$ lies on this median, so all three medians intersect at the centroid.

33. $\frac{5}{2}$ **34.** 9 **35.** 4

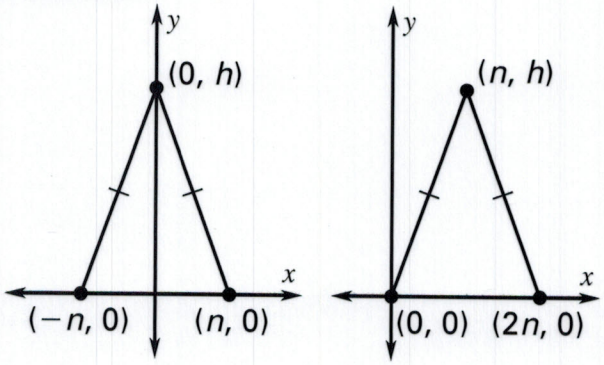
36. $\frac{9\sqrt{19}}{2}, \frac{9\sqrt{19}}{2}$; yes; the height and base of both triangles will always be the same.

5.4 Problem Solving

37. B; it is the centroid of the triangle.

40. Right Δ
Orthocenter is on the vertex at the right angle

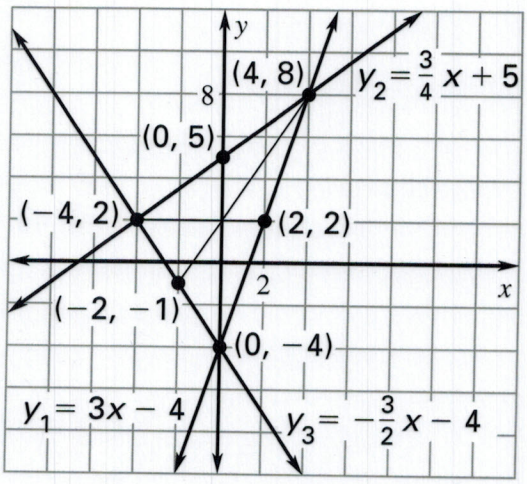
38. *Sample:*



39. 6.75 in.^{2}; median

40. Right; the orthocenter is on the right angle.

41. (0, 2);



42. a. Statements (Reasons)

1. $\triangle ABC$ is equilateral, $\overline{BD}$ is an altitude of $\triangle ABC$. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Definition of equilateral triangle)
3. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of Segment Congruence)
4. $\angle ADC$ and $\angle CDB$ are right angles. (Definition of altitude)
5. $\triangle ABD \cong \triangle CBD$ (HL)
6. $\overline{AD} \cong \overline{CD}$ (Corr. parts of $\cong$ $\triangle$ are $\cong$.)
7. $AD = CD$ (Definition of segment congruence)
8. $\overline{BD}$ is a perpendicular bisector of $\overline{AC}$. (Definition of perpendicular bisector)

b. Statements (Reasons)

1. $\triangle ABC$ is equilateral. (Given)
2. Draw a line perpendicular to $\overline{AC}$ passing through B. (Perpendicular Postulate)
3. $\overline{AB} \cong \overline{BC}$, $\angle A \cong \angle C$ (Definition of equilateral triangle)
4. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of Segment Congruence)
5. $\triangle ADB \cong \triangle CDB$ (HL)
6. $\overline{AD} \cong \overline{CD}$ (Corr. parts of $\cong$ $\triangle$ are $\cong$.)
7. $AD = CD$ (Definition of segment congruence)
8. $\overline{BD}$ is a perpendicular bisector of $\overline{AC}$. (Definition of perpendicular bisector)

43. a. Check students' work.

b. Their areas are the same.

c. They weigh the same; it means the weight of $\triangle ABC$ is evenly distributed around its centroid.

44. a. $-\frac{2}{3}$

b. $y = \frac{1}{2}x$, $x = 8$, $y = -\frac{2}{3}x + \frac{28}{3}$, $(8, 4)$

c. Find the equation of each perpendicular bisector of each side and solve the system.

45. a. Statements (Reasons)

1. $\overline{LP}$ and $\overline{MQ}$ are medians of scalene $\triangle LMN$, R is on $\overrightarrow{LP}$ such that $\overline{LP} \cong \overline{PR}$, S is on $\overrightarrow{MQ}$ such that $\overline{MQ} \cong \overline{QS}$. (Given)
2. $\overline{MP} \cong \overline{NP}$, $\overline{QL} \cong \overline{QN}$ (Definition of median)
3. $\angle MPL \cong \angle NPR$, $\angle MQL \cong \angle SQN$ (Vertical Angles Congruence Theorem)
4. $\triangle MPL \cong \triangle NPR$, $\triangle MQL \cong \triangle SQN$ (SAS)
5. $\overline{ML} \cong \overline{NR}$, $\overline{ML} \cong \overline{NS}$ (Corr. parts of $\cong$ $\triangle$ are $\cong$.)
6. $\overline{NR} \cong \overline{NS}$ (Transitive Property of Segment Congruence)

b. Statements (Reasons)

1. $\triangle MPL \cong \triangle NRP$, $\triangle MQL \cong \triangle SQN$ (Exercise 45a)
2. $\angle MLP \cong \angle NRP$, $\angle MLQ \cong \angle SQN$ (Corr. parts of $\cong$ $\triangle$ are $\cong$.)
3. $\overline{LM} \parallel \overline{RN}$, $\overline{LM} \parallel \overline{SN}$ (Converse of Alternate Interior Angles Theorem)

c. Statements (Reasons)

1. $\overline{LM} \parallel \overline{RN}$, $\overline{LM} \parallel \overline{SN}$ (Exercise 45b)
2. S, N, and R are collinear. (Parallel Postulate)

5.4 Mixed Review

46. $y = 3x + 7$

47. $y = -\frac{1}{2}x - 6$

48. $y = -5x + 4$

49. 23 **50.** $x < 11$

51. $x > -6$ **52.** $x < 4$

53. LP and LN, PM and NM

54. 2 **55.** 15

Quiz:

1) 10; ∠ Bisector Thm

2) 5; Concurrency of ∠ Bisectors of a △ Thm

3) 15 4) 18

5) 6