Answers for 5.1

For use with pages 298–301

5.1 Skill Practice

1. midsegment

2. *Sample answer:* The vertex of the right triangle is located at the origin and the other two vertices are located on the x-axis and y-axis which limits the number of variables need to label them; label the vertex on the x-axis $(2a, 0)$ and the vertex on the y-axis $(0, 2b)$.

3. 13 **4.** 10 **5.** 6

6. $\overline{YZ}$ **7.** $\overline{XZ}$ **8.** $\overline{KL}$

9. $\overline{JX}, \overline{KL}$ **10.** $\overline{XK}, \overline{KZ}$

11. $\overline{YL}, \overline{LZ}$

12–19. Sample answers are given.

12. (0, 0), (3, 0), (0, 2)

13. (0, 0), (7, 0), (0, 7)

14. (0, 0), (3, 0), (3, 3), (0, 3)

15. (0, 0), (2m, 0), (a, b)

16. (0, 0), (a, 0), (a, b), (0, b)

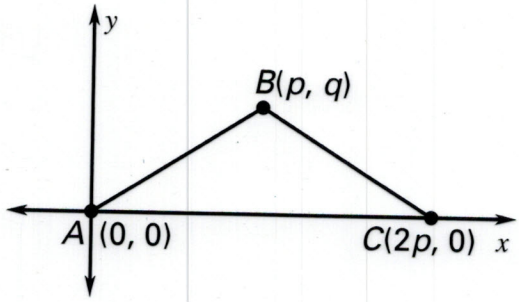
17. (0, 0), (s, 0), (s, s), (0, s)

18. (0, 0), (p, 0), (0, p)

19. (0, 0), (r, 0), (0, s)

20. $\sqrt{r^2 + s^2}$; yes

21.

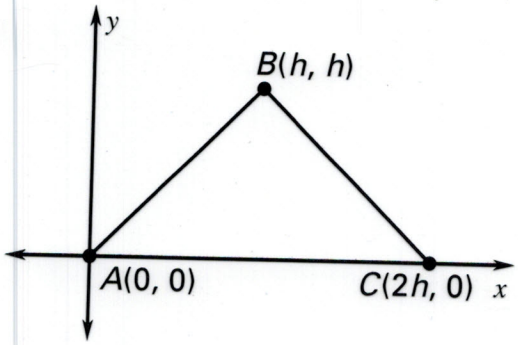


$AB = \sqrt{p^2 + q^2}, \frac{q}{p}, \left(\frac{p}{2}, \frac{q}{2}\right)$;

$BC = \sqrt{p^2 + q^2}, -\frac{q}{p}, \left(\frac{3q}{2}, \frac{q}{2}\right)$;

$CA = 2p$, 0, (p, 0); no; yes; it's not a right triangle because none of the slopes are negative reciprocals and it is isosceles because two of the sides are the same length.

22.

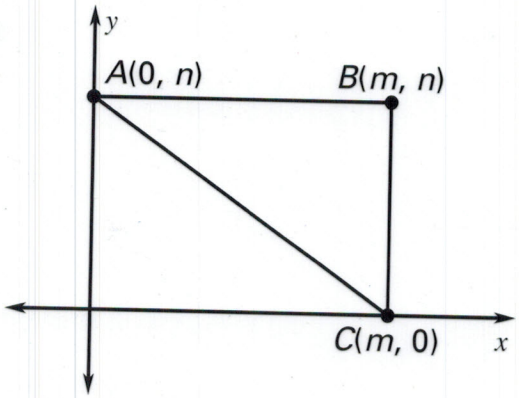


$AB = h\sqrt{2}$, 1, $\left(\frac{h}{2}, \frac{h}{2}\right)$;

$BC = h\sqrt{2}$, -1, $\left(\frac{3h}{2}, \frac{h}{2}\right)$;

$CA = 2h$, 0, (h, 0); yes; yes; it is a right triangle and an isosceles triangle since two sides are both congruent and perpendicular.

23.



$AB = m, 0, \left(\frac{m}{2}, \frac{n}{2}\right)$;

$BC = n, 0, \left(m, \frac{n}{2}\right)$;

$CA = \sqrt{m^2 + n^2}, -\frac{n}{m}, \left(\frac{m}{2}, \frac{n}{2}\right)$; yes;

no; one side is vertical and one side is horizontal thus the triangle is a right triangle. It is not isosceles since none of the sides are the same length.

24. 14 **25.** 13 **26.** 34

27. You don't know that $\overline{DE}$ and $\overline{BC}$ are parallel.

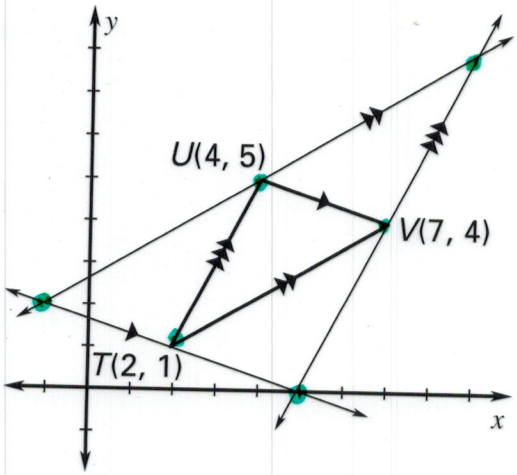
28. $PQ = 13$, $PR = 14$, $QR = 15$, perimeter $= 42$; 84

29. $(0, k)$. *Sample answer:* Since $\triangle OPQ$ and $\triangle RSQ$ are right triangles with $\overline{OP} \cong \overline{RS}$ and $\overline{PQ} \cong \overline{SQ}$, the triangles are congruent by SAS.

30. $H(-h, k)$, $G(h, k)$; slope of $\overline{HE} = -\frac{k}{3h}$ and the slope of $\overline{DG} = \frac{k}{3h}$.

31. A

32.



$(-1, 2), (5, 0), (9, 8)$

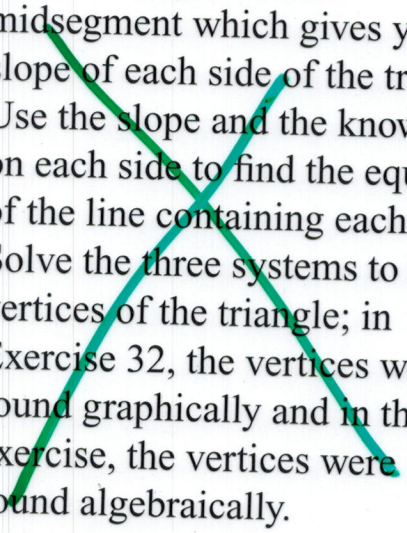
33. $GE = \frac{1}{2}DB$, $EF = \frac{1}{2}BC$,

$$\text{area of } \triangle EFG = \frac{1}{2}\left[\frac{1}{2}DB\left(\frac{1}{2}BC\right)\right]$$

$$= \frac{1}{8}(DB)(BC),$$

$$\text{area of } \triangle BCD = \frac{1}{2}(DB)(BC)$$

34. Find the slope of each midsegment which gives you the slope of each side of the triangles. Use the slope and the known point on each side to find the equation of the line containing each side. Solve the three systems to find the vertices of the triangle; in Exercise 32, the vertices were found graphically and in this exercise, the vertices were found algebraically.



5.1 Problem Solving

35. 10 ft

36. Since $PR = \sqrt{h^2 + k^2}$ and $PQ = \sqrt{h^2 + k^2}$, $\triangle PQR$ is isosceles by definition.

37. The coordinates of W are (3, 3) and the coordinates of V are (7, 3). The slope of $\overline{WV}$ is 0 and the slope of $\overline{OH}$ is 0 making $\overline{WV} \| \overline{OH}$. $WV = 4$ and $OH = 8$ thus $WV = \frac{1}{2}OH$.

38. No, no, yes, no; $AB = 4$ feet since it is half the length of $\overline{EF}$. The length of $\overline{CD}$ must be greater than 4 feet but less than 8 feet.

39. 16. *Sample answer:* $\overline{DE}$ is half the length of $\overline{FG}$ which makes $FG = 8$. $\overline{FG}$ is half the length of $\overline{AC}$ which makes $AC = 16$.

40. Definition of midsegment, Midsegment Theorem and Corresponding Angles Postulate, Midsegment Theorem and Corresponding Angles Postulate, *PST*, *SQU*, *ASA*

41. *Sample answer:* The coordinates of D are (q, r) and the coordinates of F are $(p, 0)$ since $\left(\frac{2p + 0}{2}, \frac{0 + 0}{2}\right) = (p, 0)$. The slope of $\overline{DF}$ is $\frac{r - 0}{q - p} = \frac{r}{q - p}$ and the slope of $\overline{BC}$ is $\frac{2r - 0}{2q - 2p} = \frac{r}{q - p}$, so $\overline{DF} \| \overline{BC}$. $DF = \sqrt{(p - q)^2 + r^2}$ and $BC = \sqrt{(2q - 2p)^2 + (2r)^2} = 2\sqrt{(p - q)^2 + r^2}$ making $DF = \frac{1}{2}BC$.

42. *Sample answer:* Let $A(0, 0)$, $B(0, p)$, and $D(q, 0)$ be the vertices of $\triangle ABD$. Since C is the midpoint of $\overline{BD}$, its coordinates are $\left(\frac{p}{2}, \frac{q}{2}\right)$.

$$AC = \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{2}\right)^2} = \frac{\sqrt{p^2 + q^2}}{2},$$

$$BC = \sqrt{\left(\frac{q}{2} - 0\right)^2 + \left(p - \frac{p}{2}\right)^2} = \frac{\sqrt{p^2 + q^2}}{2}, \text{ and}$$

$$DC = \sqrt{\left(\frac{q}{2} - q\right)^2 + \left(\frac{p}{2} - 0\right)^2} = \frac{\sqrt{p^2 + q^2}}{2},$$

so $AC = BC = DC$.

43. a. $\frac{1}{2}$

b. $\frac{5}{4}$

c. $\frac{19}{8}$

44. *Sample answer:* Isosceles right triangle ABC with $A(0, p)$, $B(0, 0)$, $C(p, 0)$, and right angle B.

$D\left(\frac{p}{2}, \frac{p}{2}\right)$ is the midpoint of $\overline{AC}$.

$$AB = p, BD = \frac{\sqrt{p^2 + q^2}}{2},$$

$$DA = \frac{\sqrt{p^2 + q^2}}{2}, CB = p,$$

$$DC = \frac{\sqrt{p^2 + q^2}}{2}$$ which makes the triangles congruent by SSS. In each triangle, pairs of sides are congruent making each triangle isosceles. Since $\angle CDB$ and $\angle ADB$ are a linear pair and are congruent, they are right angles, which makes the triangles congruent isosceles right triangles.

45. *Sample answer:* $\triangle ABD$ and $\triangle CBD$ are congruent isosceles right triangles with $A(0, p)$, $B(0, 0)$, $C(p, 0)$, and $D\left(\frac{p}{2}, \frac{p}{2}\right)$.

$AB = p$, $BC = p$, and $\overline{AB}$ is vertical and $\overline{BC}$ is horizontal, so $\overline{AB} \perp \overline{BC}$. By definition, $\triangle ABC$ is an isosceles right triangle.

46. *Sample answer:* A midsegment of $\overline{EN}$; the length of the quarter-segment will be $\frac{3}{4}$ the length of $\overline{LN}$ and the length of the eighth-segment will be $\frac{7}{8}$ the length of $\overline{LN}$; $\triangle LMN$, midsegment $\overline{XY}$, quarter-segment $\overline{DE}$, and eighth-segment $\overline{FG}$. $L(0, 0)$, $M(a, b)$, and $N(c, 0)$ leading to $X\left(\frac{a}{2}, \frac{b}{2}\right)$, $Y\left(\frac{a+c}{2}, \frac{b}{2}\right)$, $D\left(\frac{a}{4}, \frac{b}{4}\right)$, $E\left(\frac{a+3c}{4}, \frac{b}{4}\right)$, $F\left(\frac{a}{8}, \frac{b}{8}\right)$, and $G\left(\frac{a+7c}{8}, \frac{b}{8}\right)$. $LM = c$, $XY = \frac{c}{2}$, $DE = \frac{3}{4}c$, and $FG = \frac{7}{8}c$.

5.1 Mixed Review

47. 29 **48.** 40 **49.** 50

50. SSS; *Sample answer:* $\triangle WXY \cong \triangle WZY$

51. ASA; $\triangle ABC \cong \triangle CDA$

52. AAS; $\triangle PQR \cong \triangle RSP$