9.4 Perform Rotations

rotation - a transformation in which a figure is turned about a fixed point, called the **center of rotation**

angle of rotation - the angle formed by drawing rays from the center of rotation to a point and its image

A rotation about a point P through an angle $x°$ maps every point Q in the plane to a point Q′ so that one of the following properties is true:

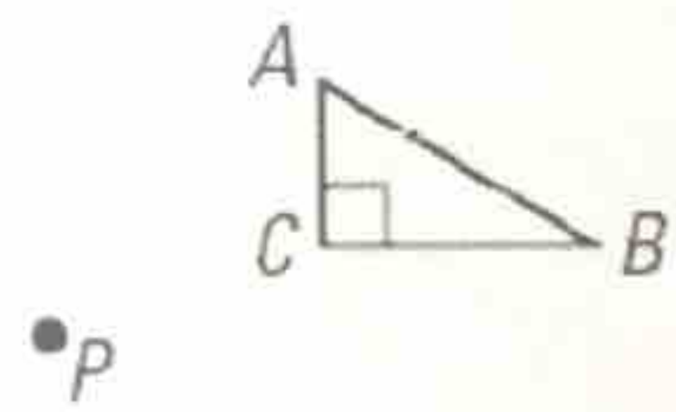
* If Q is not the center of rotation P, then $QP = Q'P$ and $m\angle QPQ' = x°$,

  or

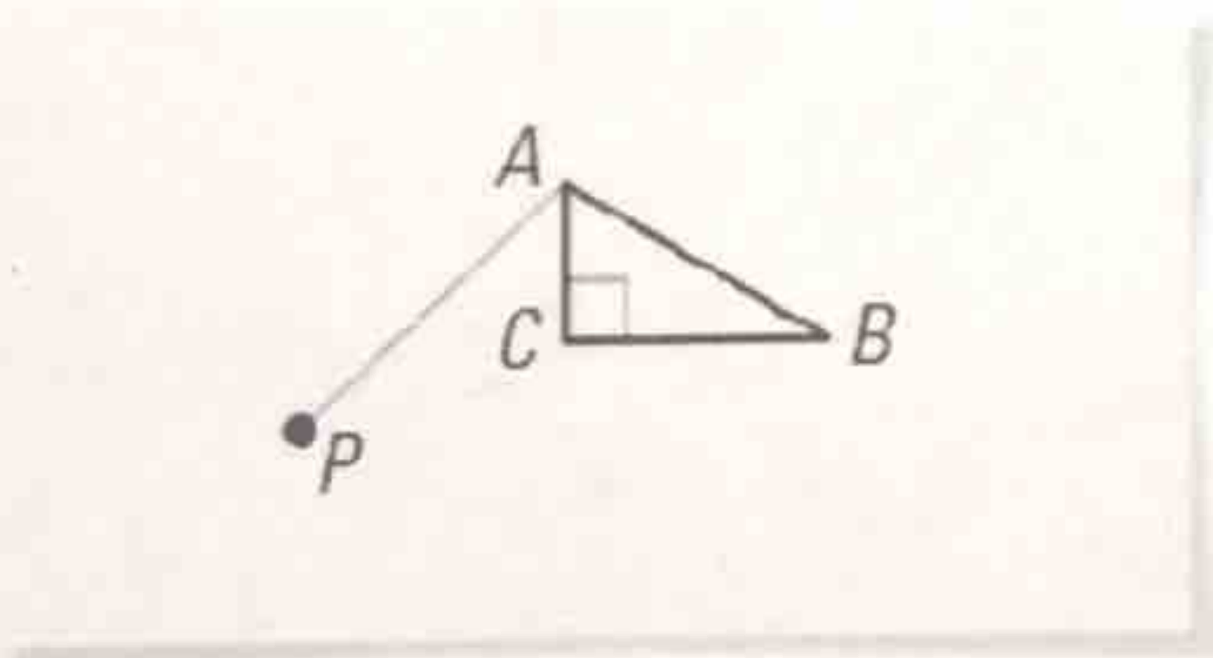
* If Q is the center of rotation P, then the image of Q is Q

Note: in this chapter, all rotations will be counterclockwise

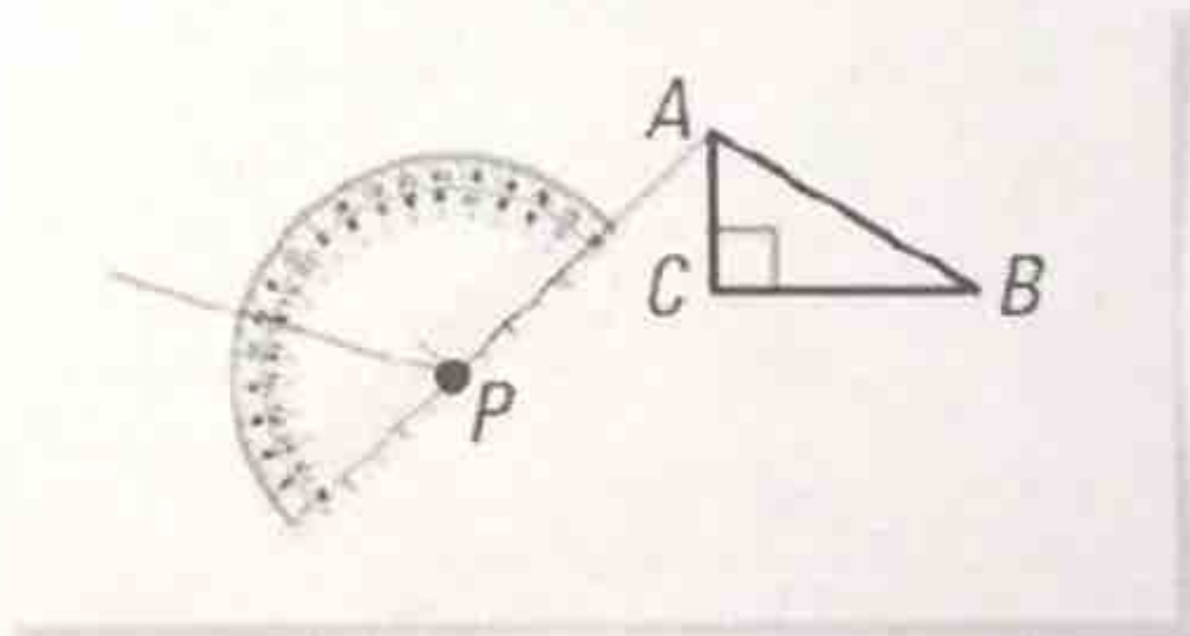
Ex 1: Draw a 120° rotation of $\triangle ABC$ about P.



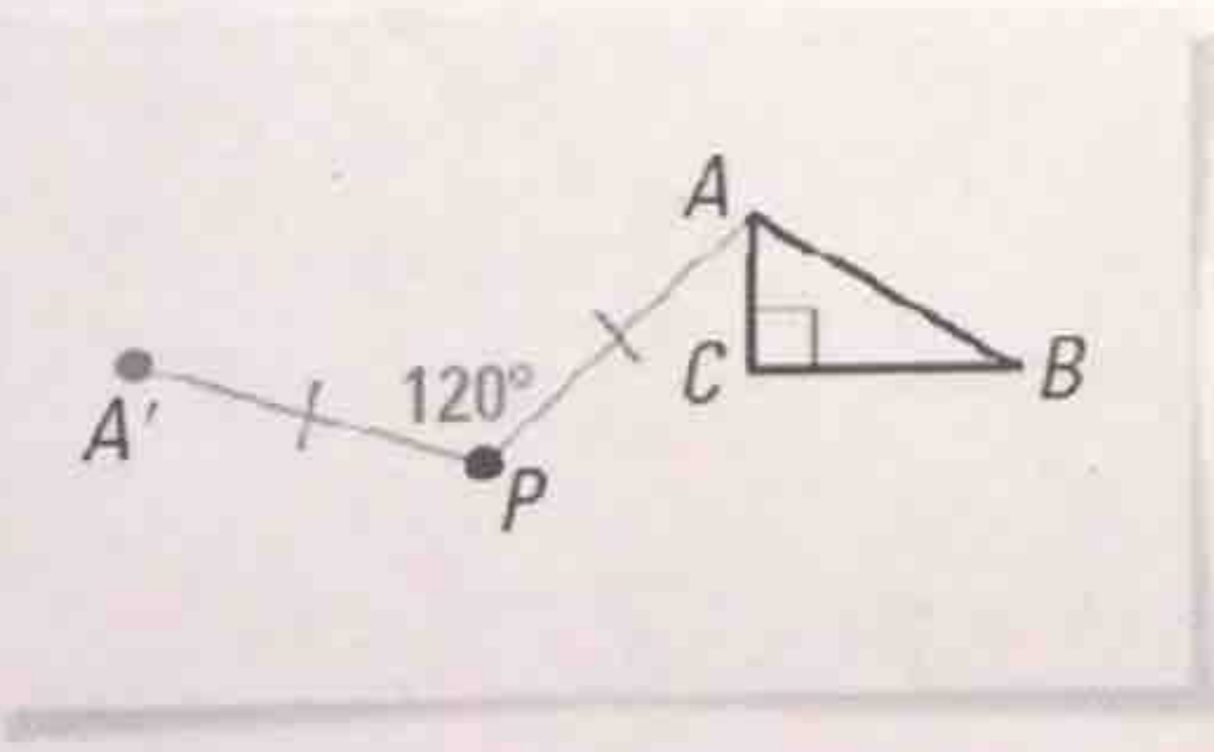
STEP 1 **Draw** a segment from A to P.



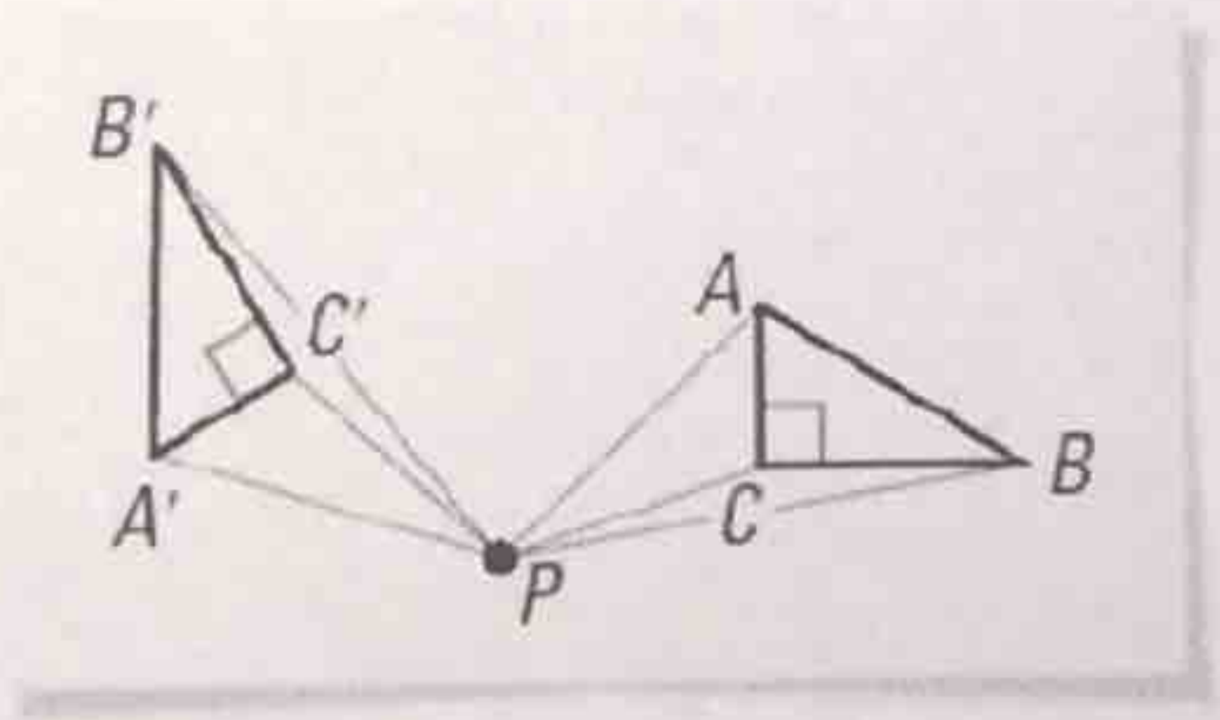
STEP 2 **Draw** a ray to form a 120° angle with $\overline{PA}$.



STEP 3 **Draw** A' so that $PA' = PA$.



STEP 4 **Repeat** Steps 1–3 for each vertex. Draw $\triangle A'B'C'$.

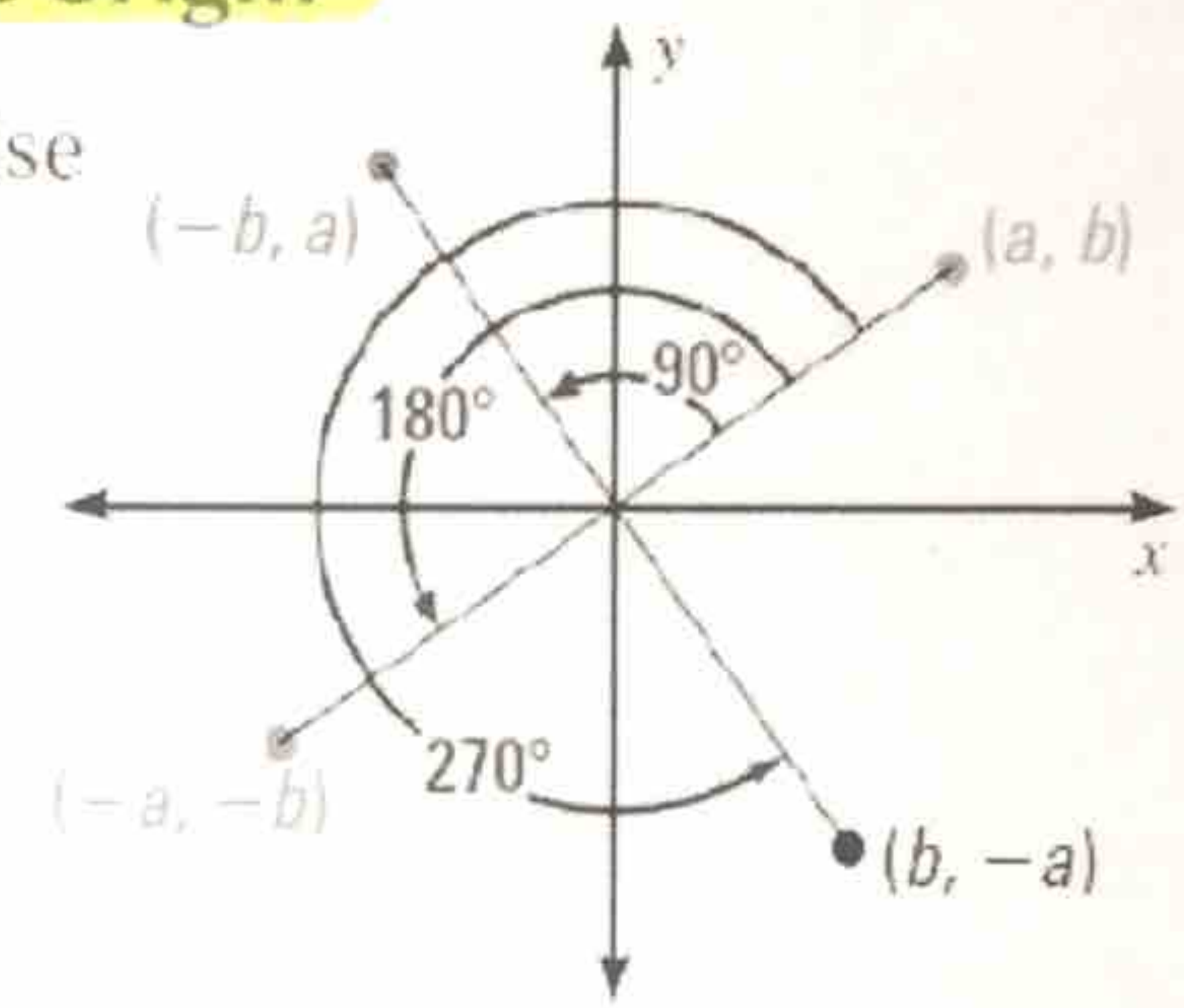


KEY CONCEPT *For Your Notebook*

Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true:

1. For a rotation of 90°, $(a, b) \rightarrow (-b, a)$.
2. For a rotation of 180°, $(a, b) \rightarrow (-a, -b)$.
3. For a rotation of 270°, $(a, b) \rightarrow (b, -a)$.



Ex 2: Graph quadrilateral RSTU with vertices R(3, 1), S(5, 1), T(5, -3), and U(2, -1). Then rotate the quadrilateral 270° about the origin.

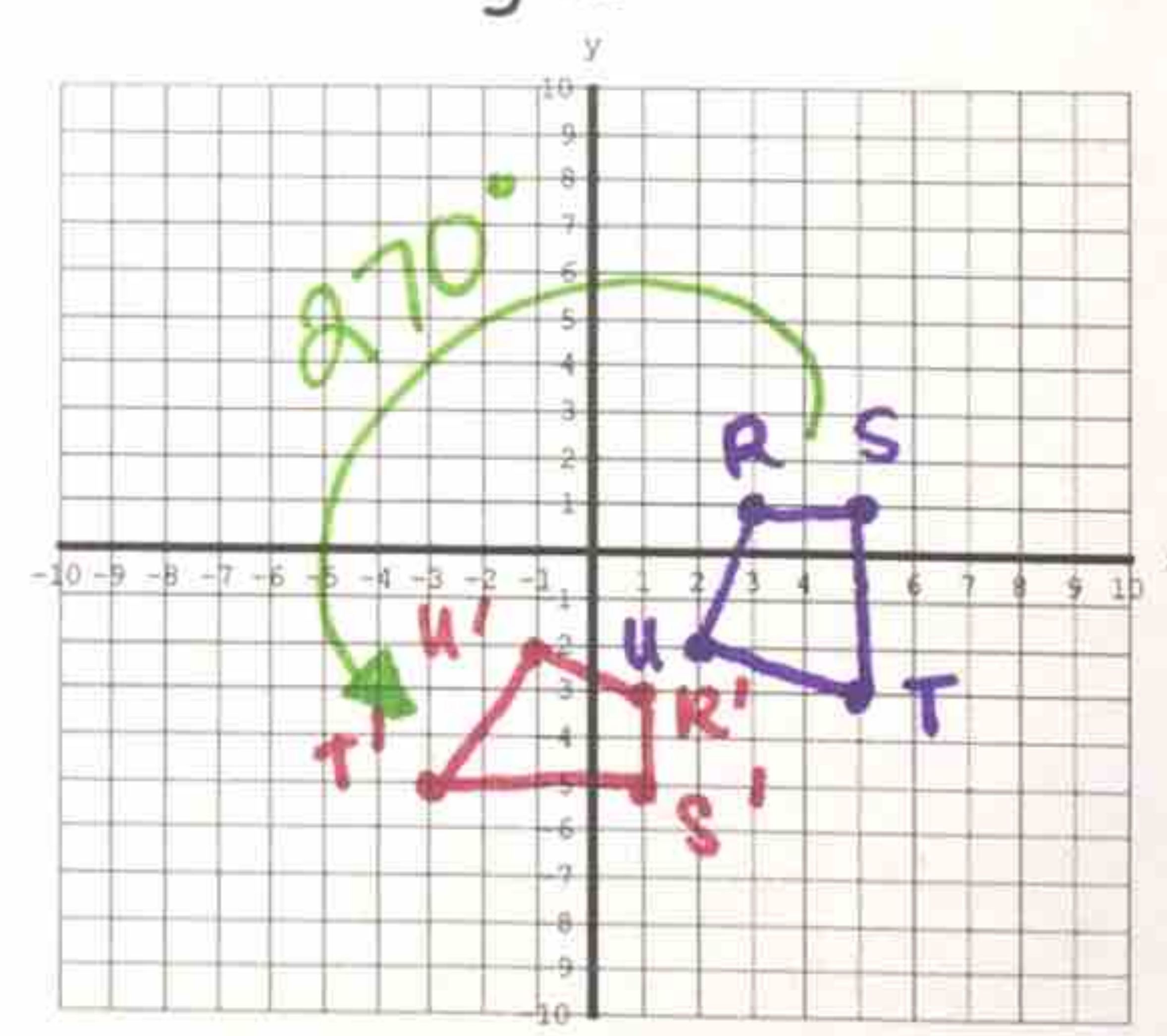
$(x, y) \rightarrow (y, -x)$

$R(3,1) \rightarrow R'(1,-3)$

$S(5,1) \rightarrow S'(1,-5)$

$T(5,-3) \rightarrow T'(-3,-5)$

$U(2,-1) \rightarrow U'(-1,-2)$

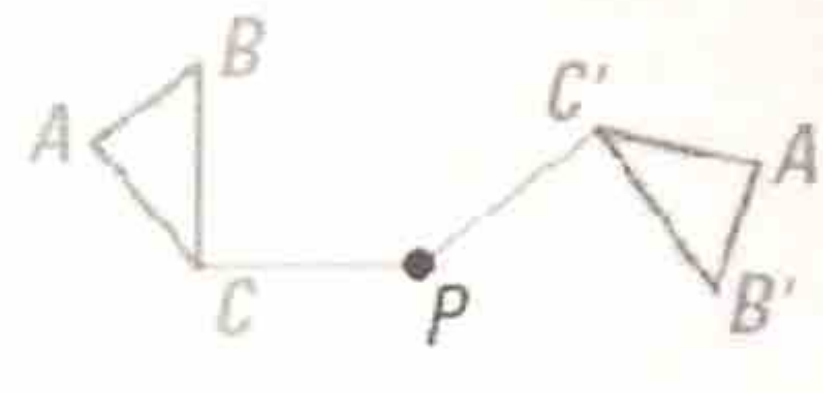


THEOREM *For Your Notebook*

THEOREM 9.3 Rotation Theorem

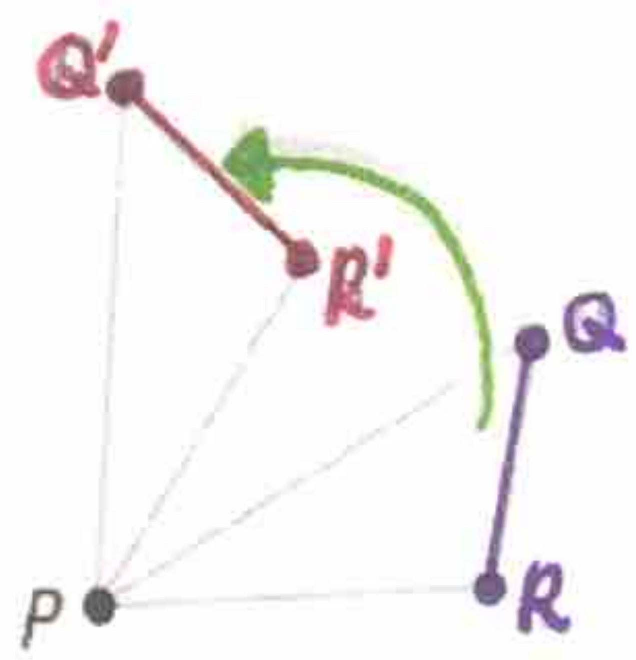
A rotation is an isometry.

Proof: Exs. 33–35, p. 604

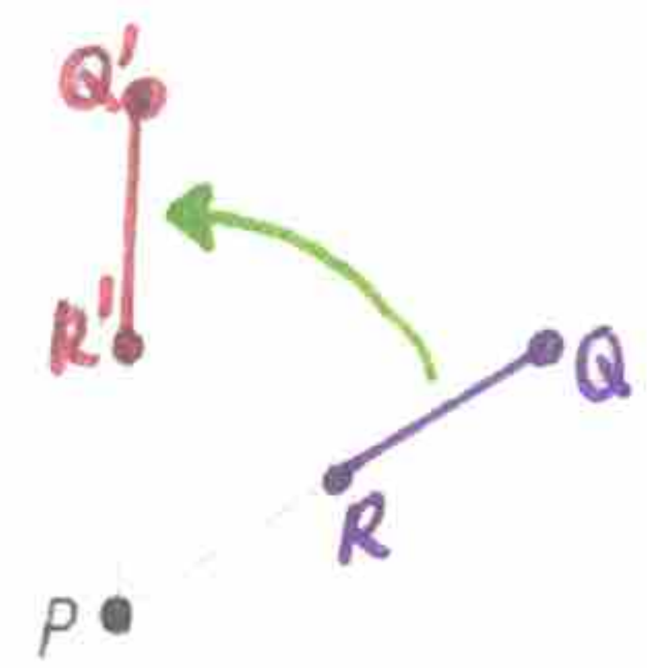


$\triangle ABC \cong \triangle A'B'C'$

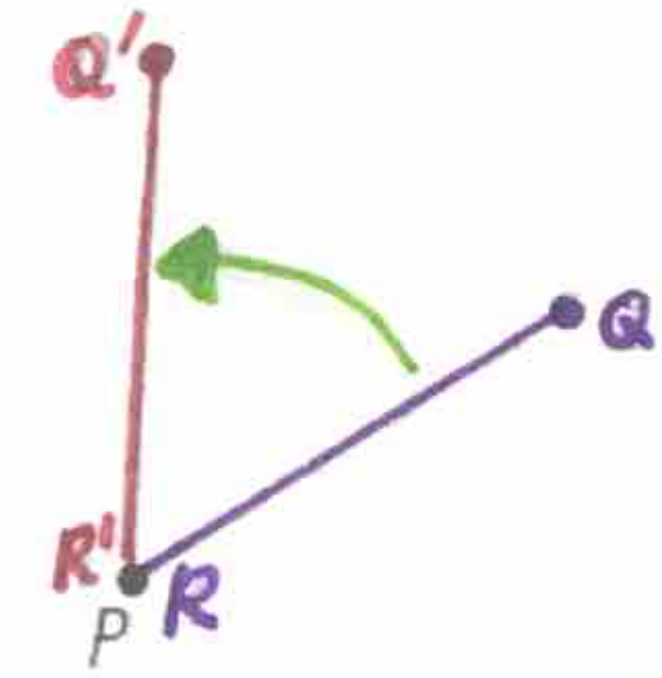
To prove the Rotation Theorem, you need to show that a rotation preserves the length of a segment. There are THREE cases to consider:



Case 1 *R*, *Q*, and *P* are noncollinear.

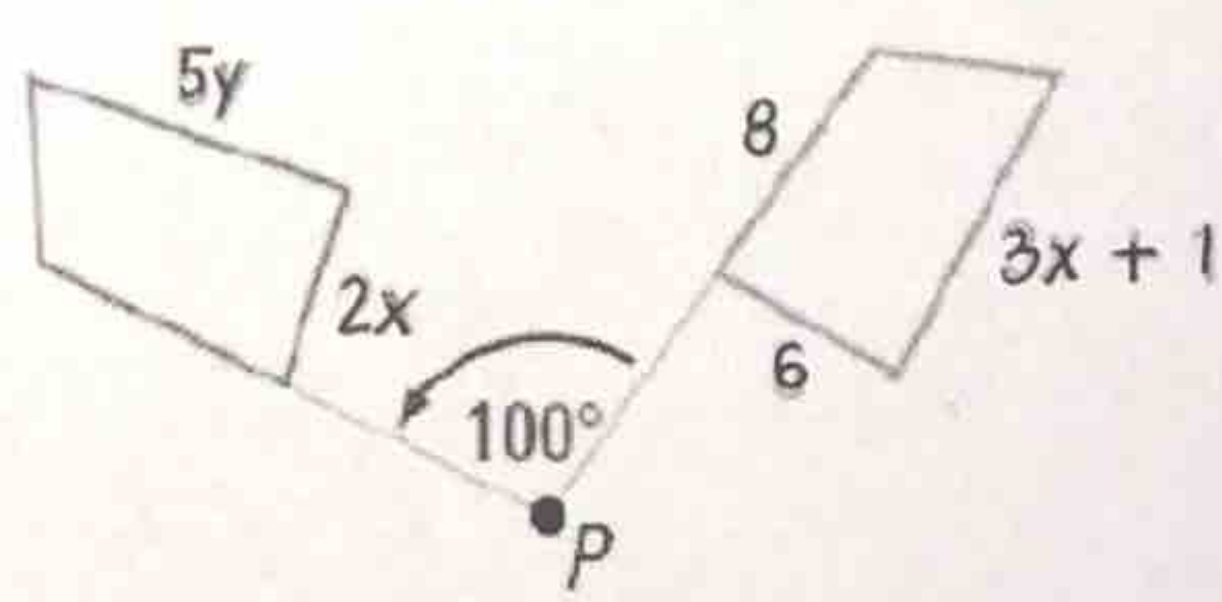


Case 2 *R*, *Q*, and *P* are collinear.



Case 3 *P* and *R* are the same point.

<u>Ex 3</u>: The quadrilateral is rotated about P. What is the value of y?



Rotation is an isometry

so

$2x = 6$

$x = 3$

$5y = 3x + 1$

$5y = 3(3) + 1$

$5y = 9 + 1$

$5y = 10$

$\boxed{y = 2}$