9.3 Perform Reflections

reflection – a transformation that uses a line as a mirror to reflect an image, this line is the **line of reflection**.

A reflection in the line **m** maps the point P in the plane to a point P′, so that for each point one of the following properties is true:

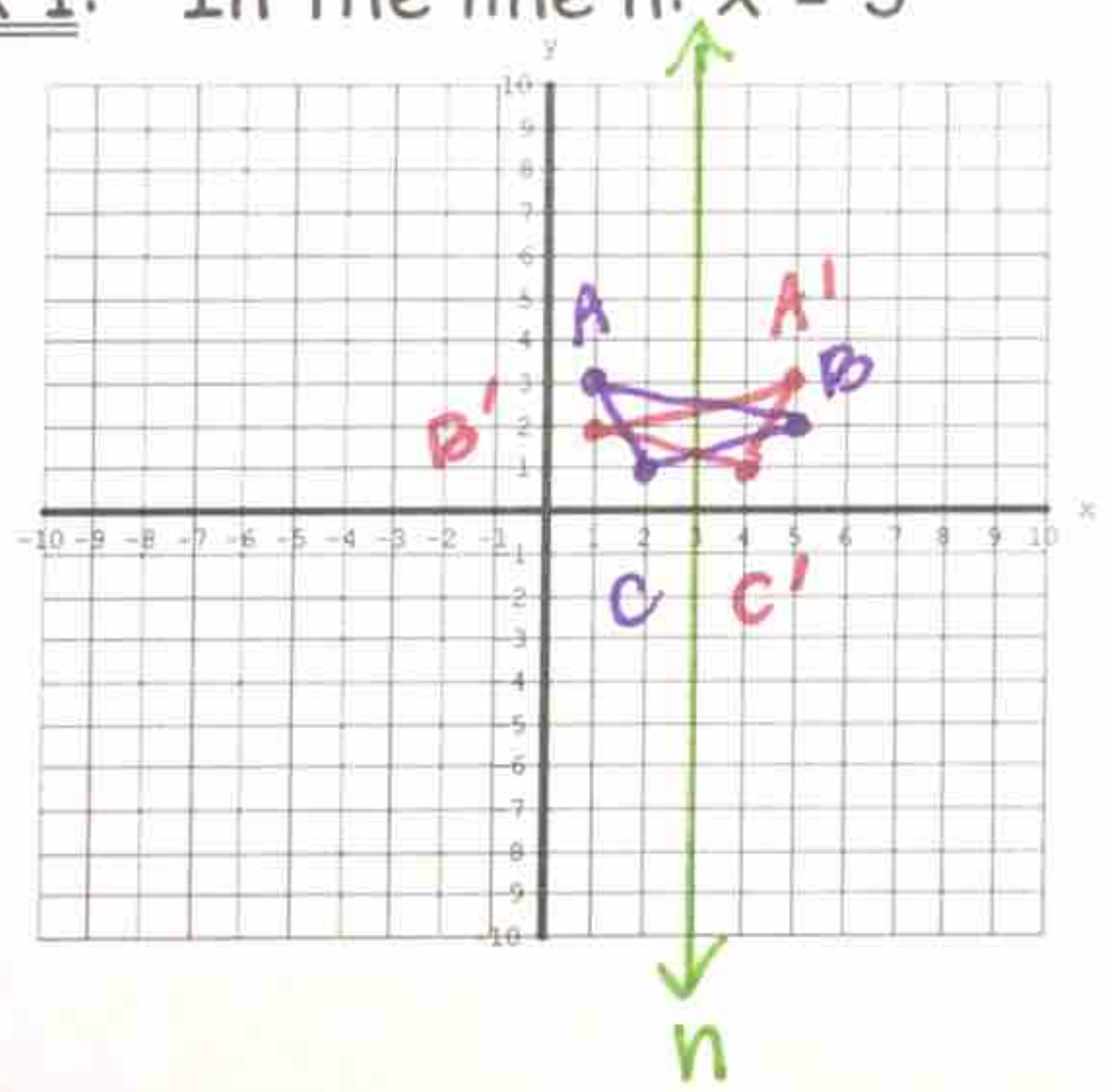
- * If P is not on the line **m**, then **m** is the perpendicular bisector of $\overline{PP'}$

  or

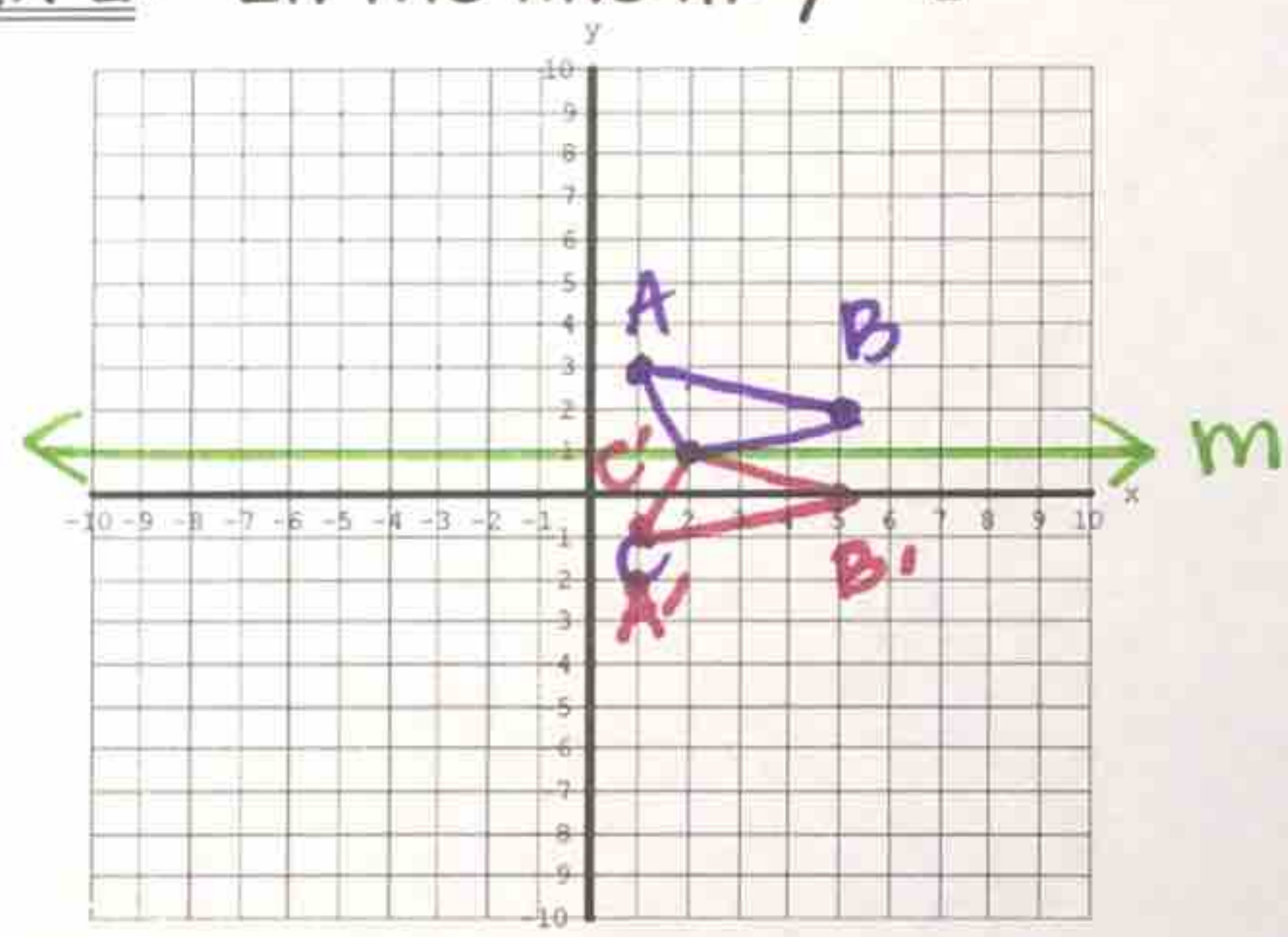
- * If P is on the line **m**, then P = P′

The vertices of ΔABC are A(1, 3), B(5, 2), and C(2, 1). Graph the reflection of ΔABC described.

Ex 1: In the line n: x = 3



Ex 2: In the line m: y = 1



KEY CONCEPT *For Your Notebook*

Coordinate Rules for Reflections

- If (a, b) is reflected in the x-axis, its image is the point $(a, -b)$.
- If (a, b) is reflected in the y-axis, its image is the point $(-a, b)$.
- If (a, b) is reflected in the line $y = x$, its image is the point (b, a).
- If (a, b) is reflected in the line $y = -x$, its image is the point $(-b, -a)$.

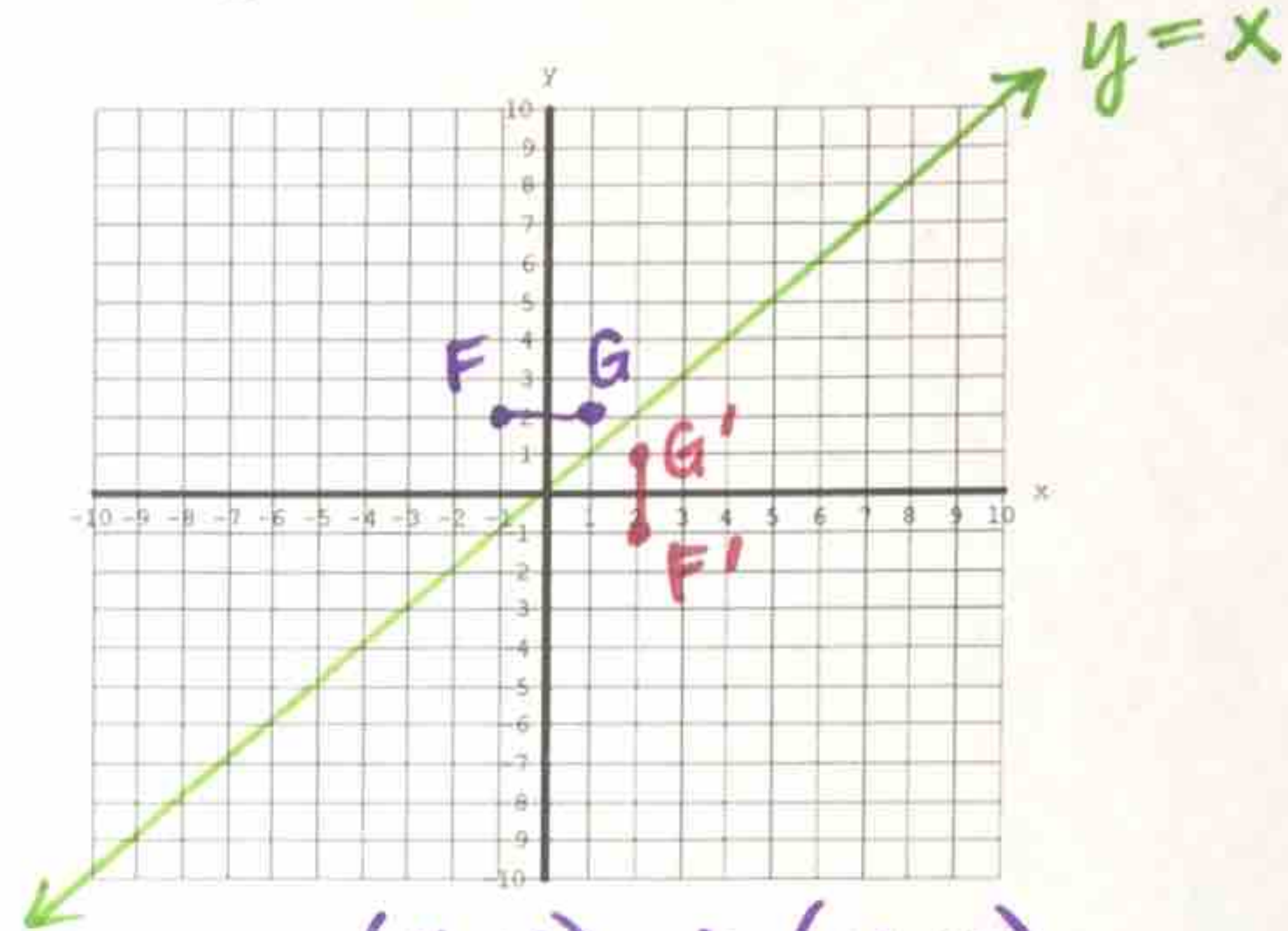
<u>Ex 3</u>: The endpoints of $\overline{FG}$ is F(-1, 2) and G(1, 2). Reflect the segment in the line y = x. Graph the segment and its image.

The slope of $y=x$ is $m=1$

so $m_{FF'}=-1$

and $m_{GG'}=-1$

because they will be $\perp$ to $y=x$.



$(x,y)\rightarrow(y,x)$

$F(-1,2)\rightarrow F'(2,-1)$

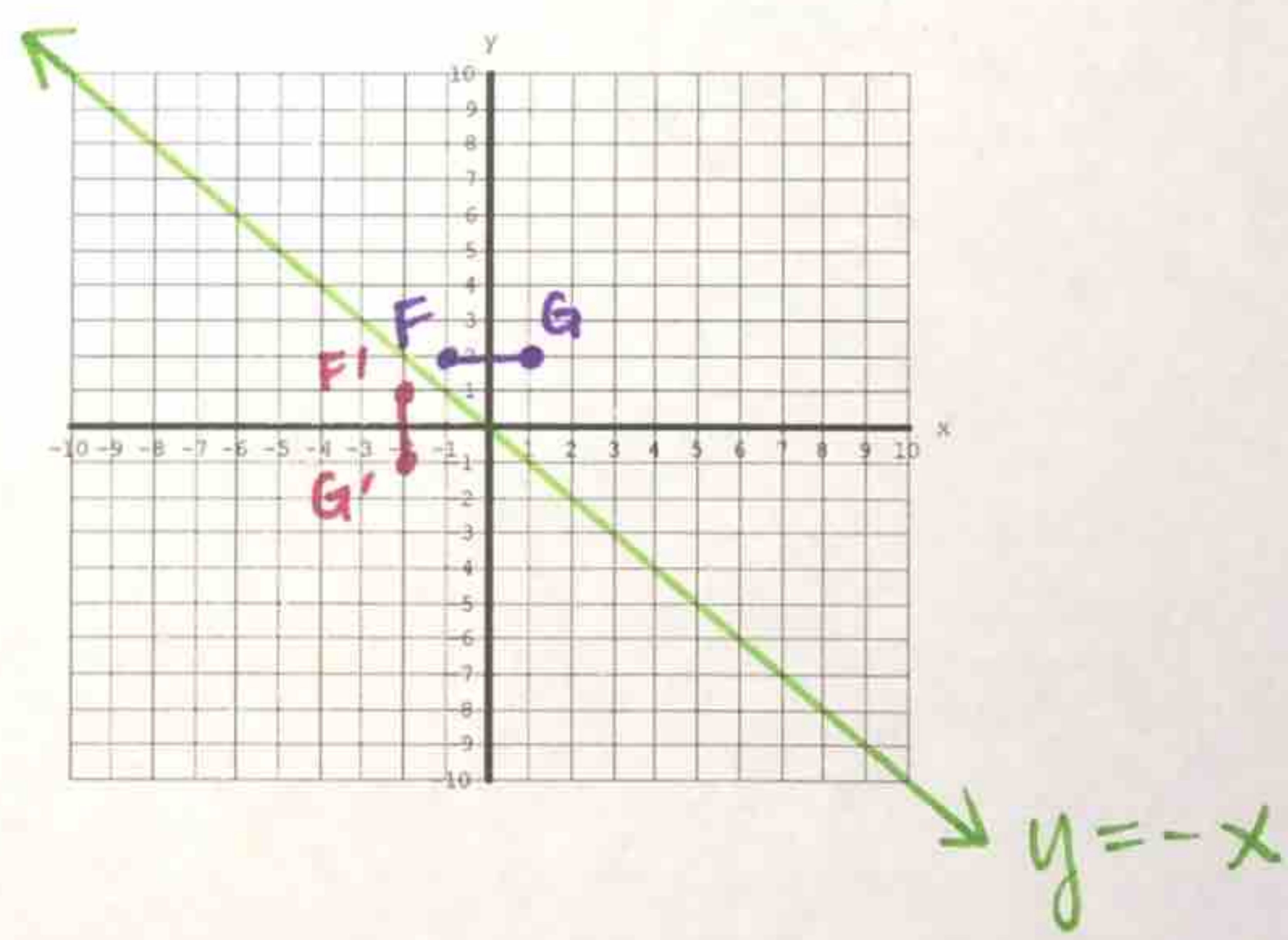
$G(1,2)\rightarrow G'(2,1)$

<u>Ex 4</u>: Reflect $\overline{FG}$ in the line y = -x. Graph the segment and its image.

$(x,y)\rightarrow(-y,-x)$

$F(-1,2)\rightarrow F'(-2,1)$

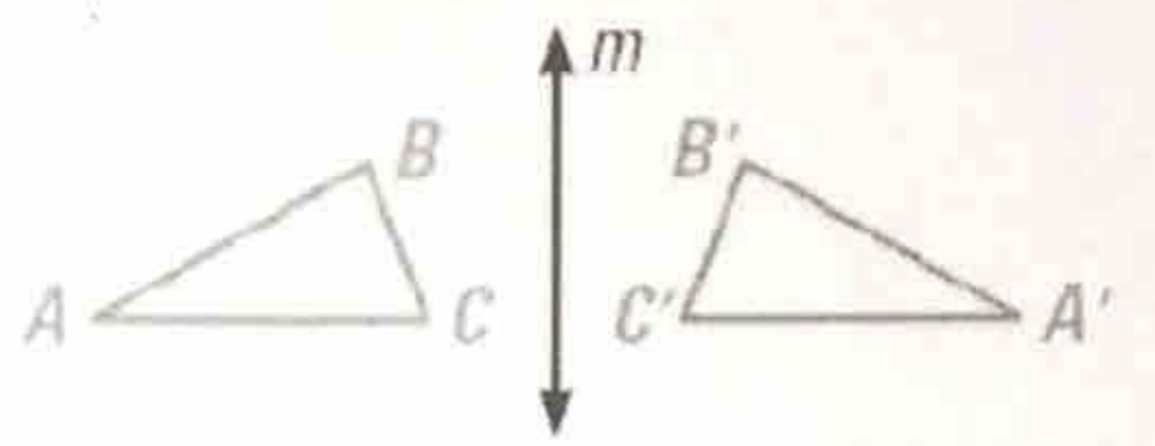
$G(1,2)\rightarrow G'(-2,-1)$



THEOREM **For Your Notebook**

THEOREM 9.2 Reflection Theorem

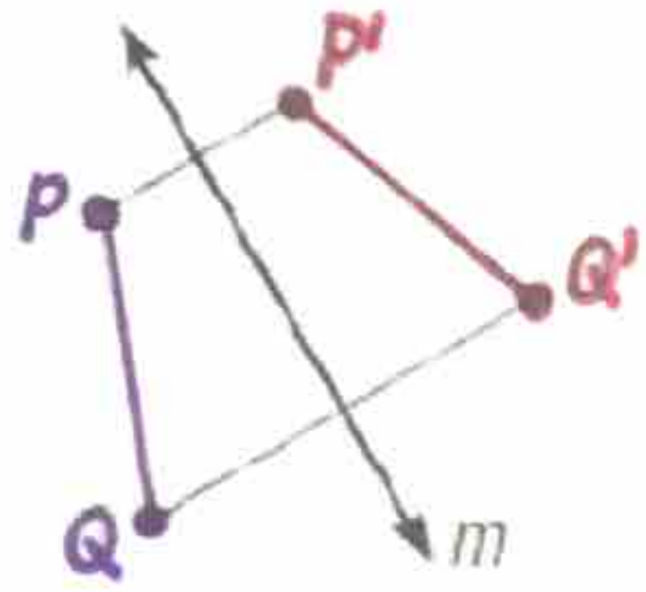
A reflection is an isometry.



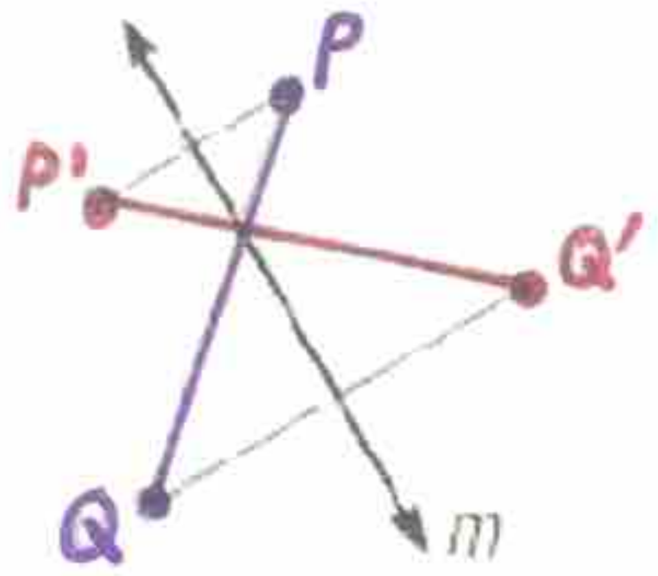
$\triangle ABC \cong \triangle A'B'C'$

Proof: Exs. 35–38, p. 595

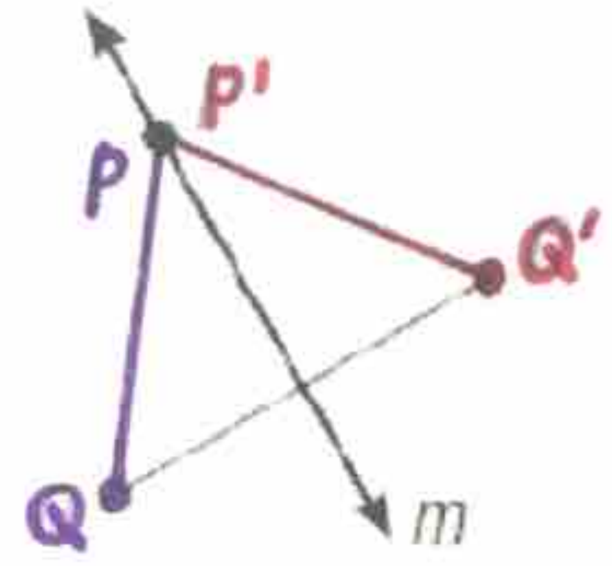
To prove the Reflection Theorem, you need to show that a reflection preserves the length of the segment. There are FOUR cases to consider:



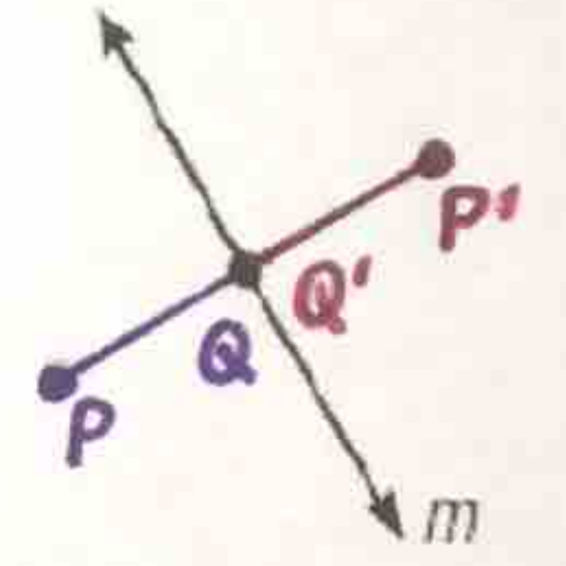
Case 1 P and Q are on the same side of m.



Case 2 P and Q are on opposite sides of m.



Case 3 P lies on m, and $\overline{PQ}$ is not $\perp$ to m.



Case 4 Q lies on m, and $\overline{PQ} \perp m$.

Ex 5: You are going to buy books. Your friend is going to buy CDs. Where should you park to minimize the distance you both will walk?

Reflect B in line m to obtain B'
Draw $\overline{CB'}$, label the intersection I
Since CB' is the shortest distance between C and B' and BC=B'C, park at point I to minimize the combined distance CI+BI, you both have to walk.