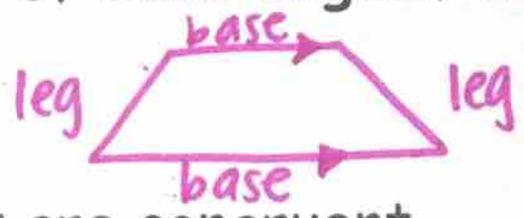
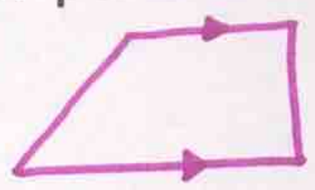
8.5 Use Properties of Trapezoids and Kites

trapezoid – a quadrilateral with exactly one pair of parallel sides, these sides are called **bases**; trapezoids have two pairs of **base angles**; the non-parallel sides are the **legs** of the trapezoid





isosceles trapezoid- a trapezoid whose legs are congruent

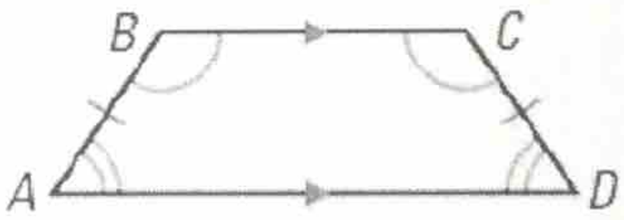
THEOREMS — For Your Notebook

THEOREM 8.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof: Ex. 37, p. 548

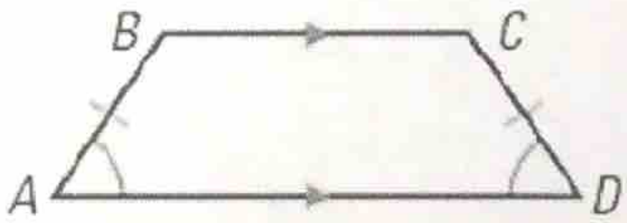


THEOREM 8.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof: Ex. 38, p. 548

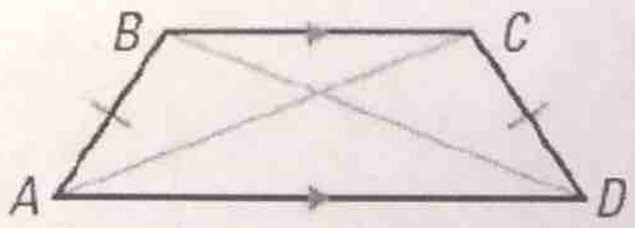


THEOREM 8.16

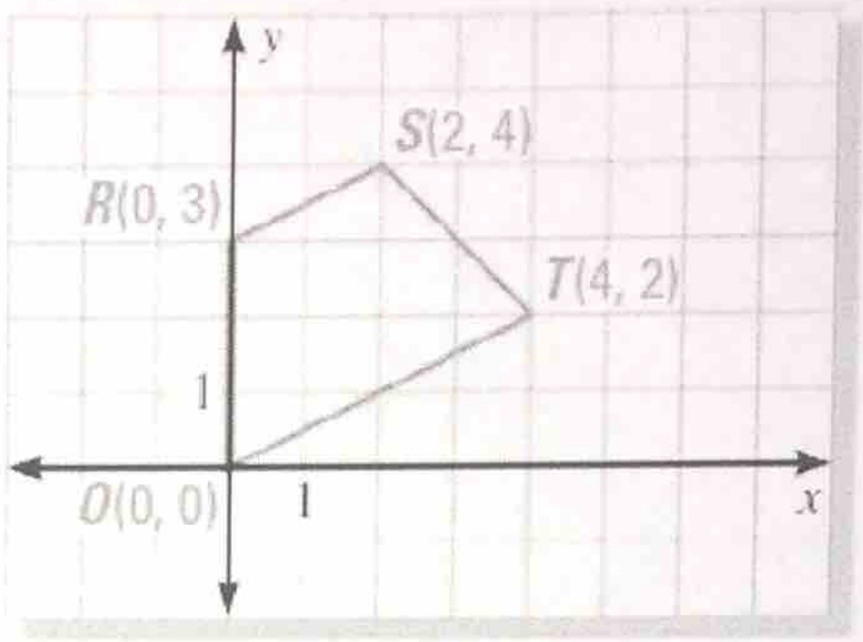
A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof: Exs. 39 and 43, p. 549



Ex 1: Show that ORST is a trapezoid.



$m_{RS} = \frac{4-3}{2-0} = \frac{1}{2}$

$m_{OT} = \frac{2-0}{4-0} = \frac{1}{2}$

so $\overline{RS} \parallel \overline{OT}$

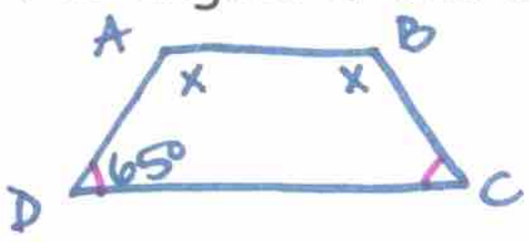
$m_{ST} = \frac{2-4}{4-2} = -1$

$m_{OR} = \frac{3-0}{0-0}$ = undefined

so $\overline{ST}$ is not parallel to $\overline{OR}$

exactly one pair of parallel sides, so trapezoid

Ex 2: The top of a table is an isosceles trapezoid. Find the measure of all the angles if one of the angles measure 65°.



$m\angle D = m\angle C = 65°$

$x + x + 65 + 65 = 360$

$2x + 130 = 360$

$2x = 230$

$x = 115$

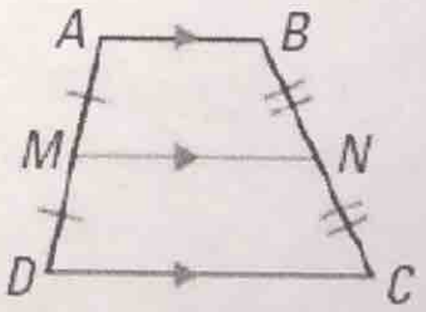
$m\angle A = m\angle B = 115°$

midsegment of a trapezoid - the segment that connects the midpoints of the legs of a trapezoid

THEOREM *For Your Notebook*

THEOREM 8.17 Midsegment Theorem for Trapezoids

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.



If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Justification: Ex. 40, p. 549
Proof: p. 937

kite - a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent

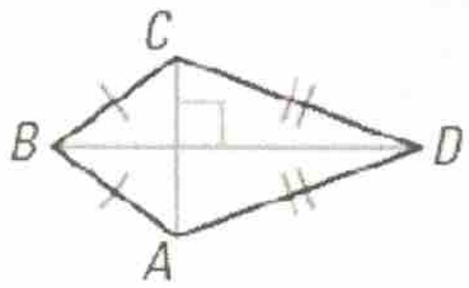
THEOREMS *For Your Notebook*

THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof: Ex. 41, p. 549

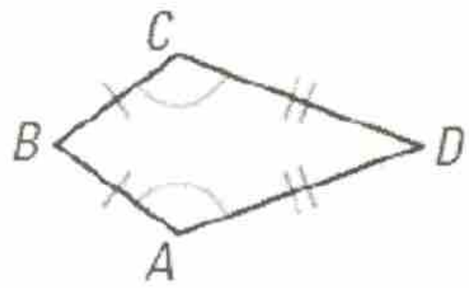


THEOREM 8.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof: Ex. 42, p. 549



Ex 3: Find m ∠ D.

$m\angle D = m\angle F = x$

$2x + 124 + 80 = 360$

$2x + 204 = 360$

$2x = 156$

$x = 78$

$\boxed{m\angle D = 78^\circ}$