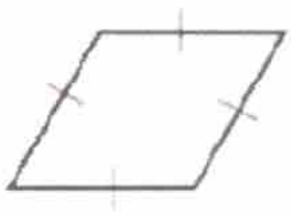
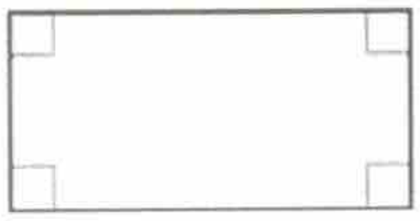
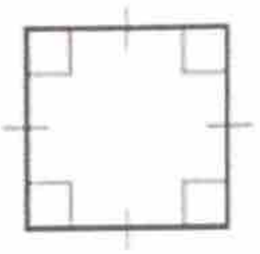
8.4 Properties of Rhombuses, Rectangles, and Squares



A rhombus is a parallelogram with four congruent sides.



A rectangle is a parallelogram with four right angles.



A square is a parallelogram with four congruent sides and four right angles.

*a square is **both** a rhombus AND a rectangle! *

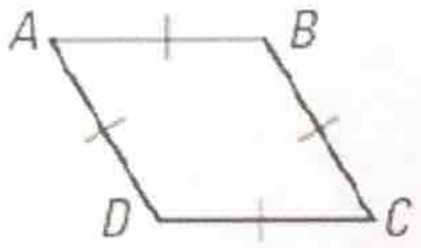
biconditional - conditional and converse are BOTH true, must prove both

COROLLARIES — For Your Notebook

Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

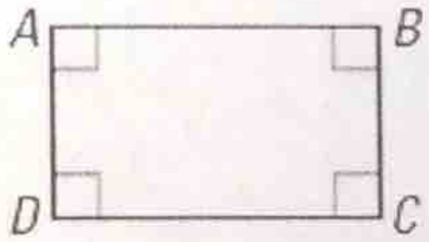


Proof: Ex. 57, p. 539

Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

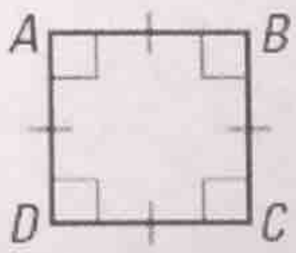


Proof: Ex. 58, p. 539

Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



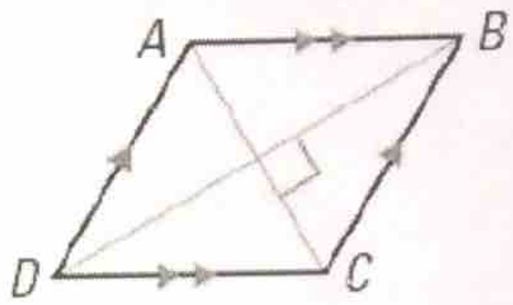
Proof: Ex. 59, p. 539

THEOREMS — For Your Notebook

THEOREM 8.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

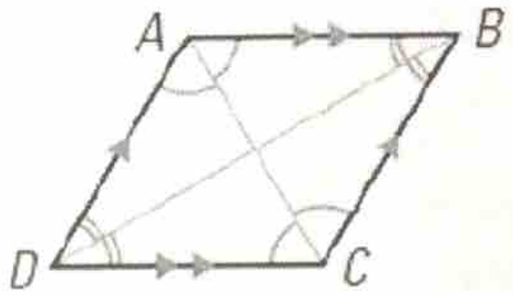


Proof: p. 536; Ex. 56, p. 539

THEOREM 8.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$ and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

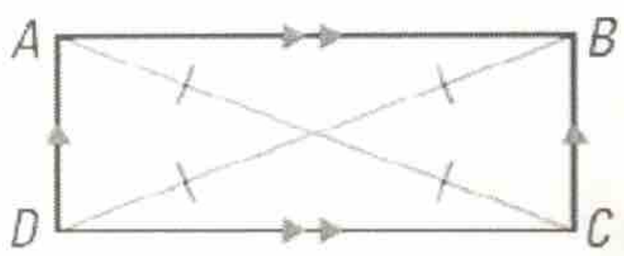


Proof: Exs. 60–61, p. 539

THEOREM 8.13

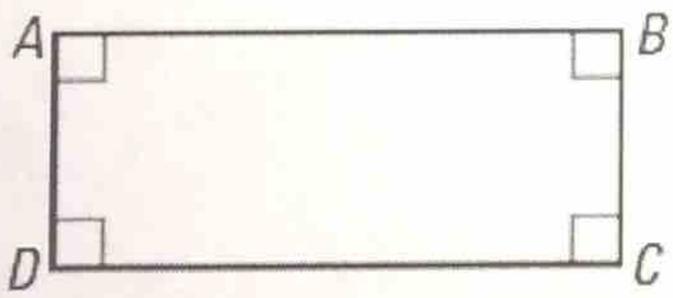
A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.



Proof: Exs. 63–64, p. 540

Ex 1: Sketch a rectangle ABCD and list all of its properties.

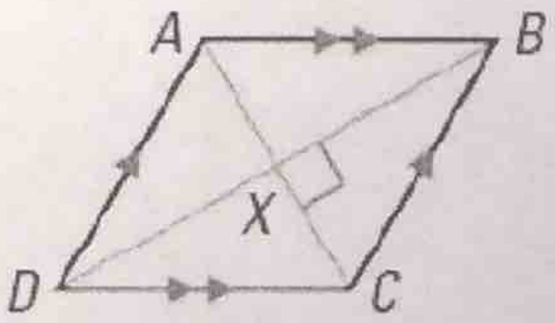


- 4 right angles
- parallelogram:
  - opposite sides are // and ≅
  - opposite angles are ≅, consec. ∠s are supp.
  - diagonals bisect each other

Ex 2: Write a paragraph proof to prove Theorem 8.11.

GIVEN ▶ $ABCD$ is a parallelogram; $\overline{AC} \perp \overline{BD}$

PROVE ▶ $ABCD$ is a rhombus.



ABCD is a parallelogram, so $\overline{AC}$ and $\overline{BD}$ bisect each other, and $\overline{BX} \cong \overline{DX}$. Also, $\angle BXC$ and $\angle CXD$ are ≅ right angles, and $\overline{CX} \cong \overline{CX}$ by Reflexive Property. So, $\triangle BXC \cong \triangle DXC$ by the SAS Congruence Postulate. By CPCTC, $\overline{BC} \cong \overline{DC}$. Opposite sides of $\square ABCD$ are congruent, so $\overline{AD} \cong \overline{BC} \cong \overline{DC} \cong \overline{AB}$. By definition, ABCD is a rhombus.