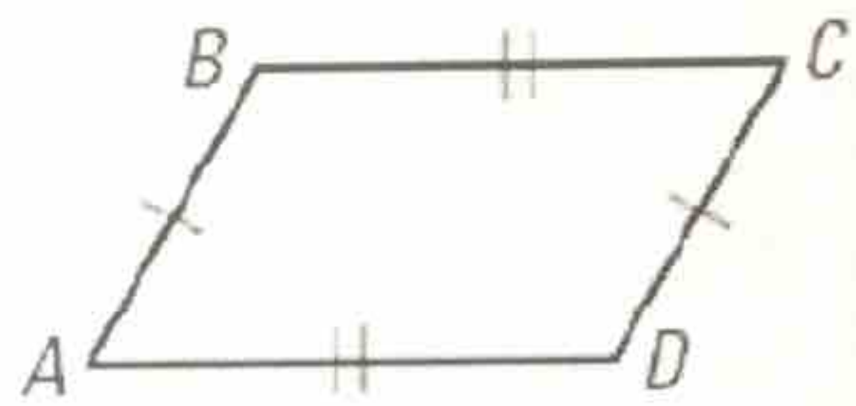
8.3 Show that a Quadrilateral is a Parallelogram

parallelogram – quadrilateral with both pairs of opposite sides parallel

THEOREMS *For Your Notebook*

THEOREM 8.7

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

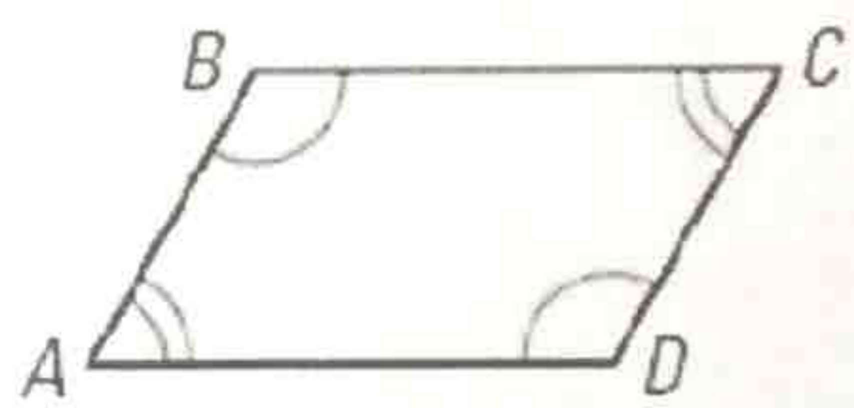


If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof: below

THEOREM 8.8

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



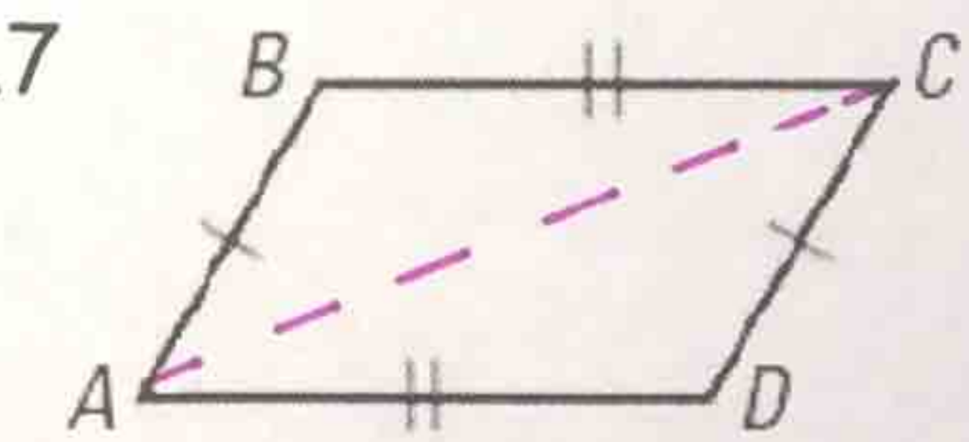
If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof: Ex. 38, p. 529

Ex 1: Write a paragraph proof to prove Theorem 8.7

GIVEN ▶ $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$

PROVE ▶ $ABCD$ is a parallelogram.

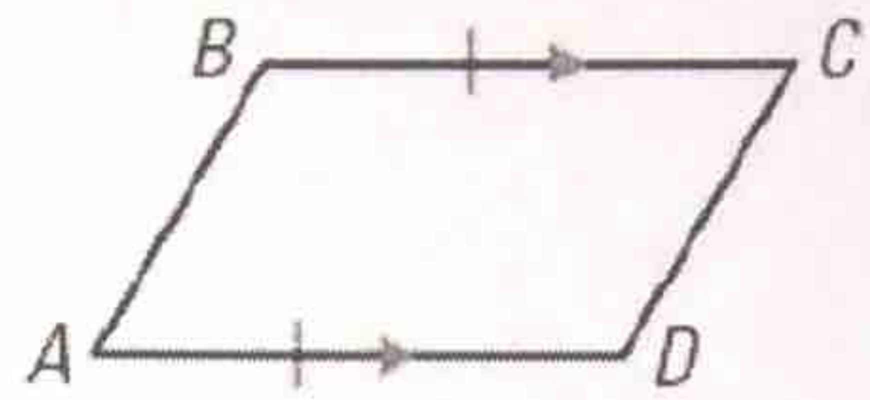


Draw $\overline{AC}$, forming $\triangle ABC$ and $\triangle CDA$. You are given that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Also, $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. So, $\triangle ABC \cong \triangle CDA$ by the SSS Congruence Postulate. Because corresponding parts of congruent triangles are congruent (CPCTC), $\angle BAC \cong \angle DCA$ and $\angle BCA \cong \angle DAC$. Then, by Alternate Interior Angles Converse, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. By definition, $ABCD$ is a parallelogram.

THEOREMS *For Your Notebook*

THEOREM 8.9

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

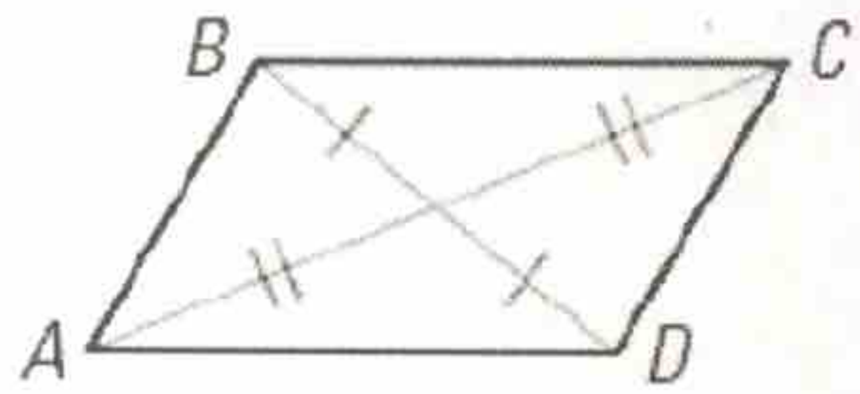


If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof: Ex. 33, p. 528

THEOREM 8.10

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



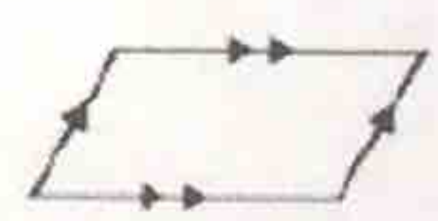
If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.

Proof: Ex. 39, p. 529

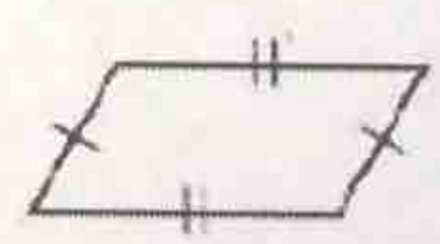
CONCEPT SUMMARY *For Your Notebook*

Ways to Prove a Quadrilateral is a Parallelogram

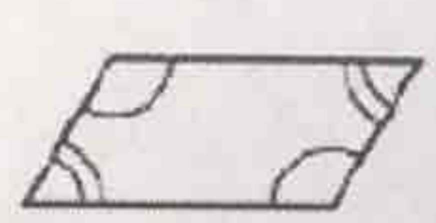
1. Show both pairs of opposite sides are parallel.
   (DEFINITION)



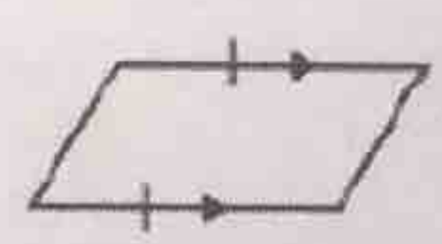
2. Show both pairs of opposite sides are congruent.
   (THEOREM 8.7)



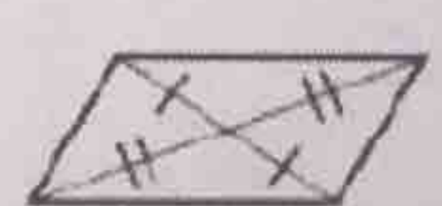
3. Show both pairs of opposite angles are congruent.
   (THEOREM 8.8)



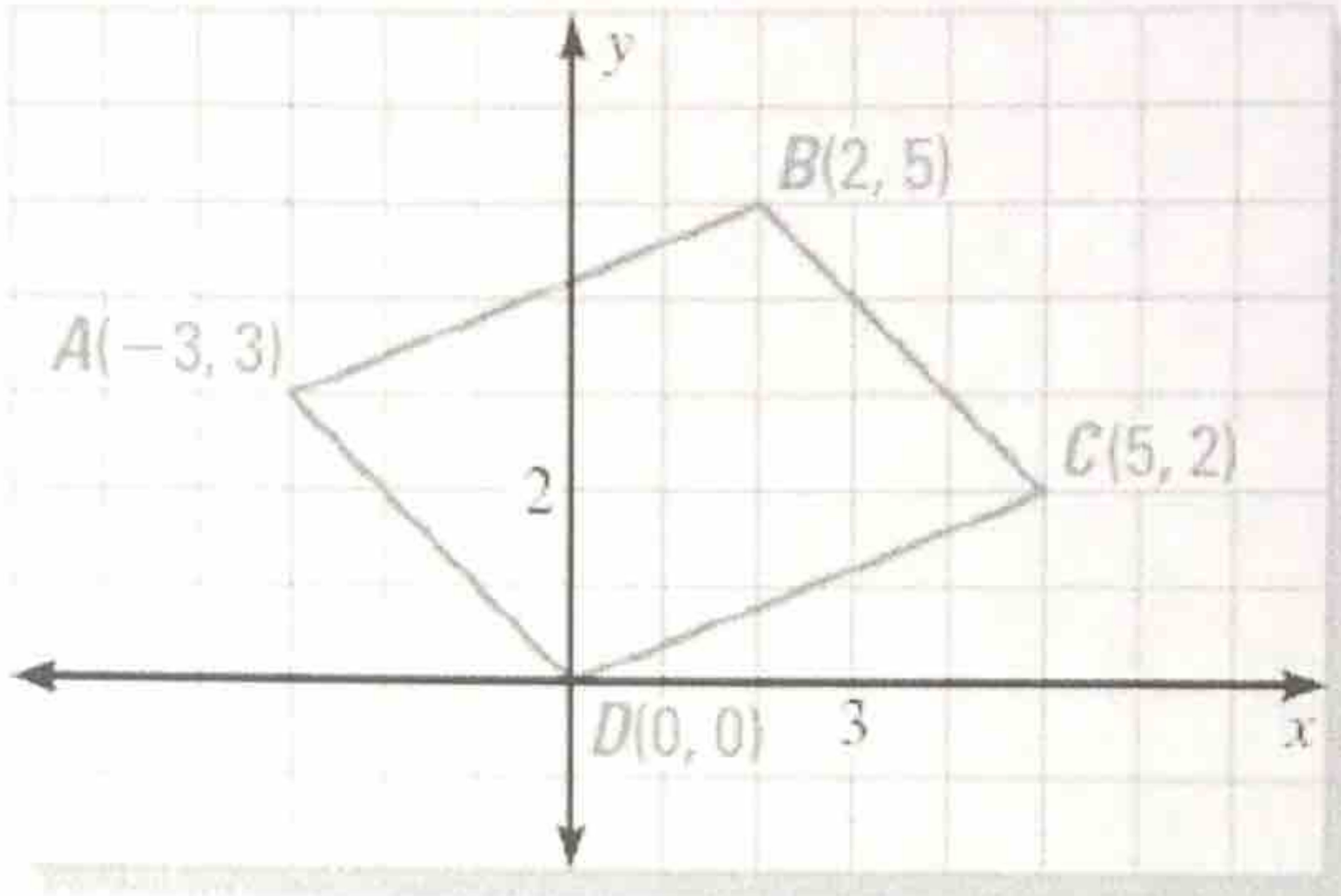
4. Show one pair of opposite sides are congruent and parallel.
   (THEOREM 8.9)



5. Show the diagonals bisect each other.
   (THEOREM 8.10)



Ex 2: Show that quadrilateral ABCD is a parallelogram in **TWO** different way.



Distance Formula:

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB=\sqrt{(2-(-3))^2+(5-3)^2}=\sqrt{29}$

$CD=\sqrt{(5-0)^2+(2-0)^2}=\sqrt{29}$

so $AB=CD$, so $\overline{AB}\cong\overline{CD}$

Slope Formula:

$m=\frac{y_2-y_1}{x_2-x_1}$

$m_{AB}=\frac{5-3}{2-(-3)}=\frac{2}{5}$

$m_{CD}=\frac{2-0}{5-0}=\frac{2}{5}$

same slope so $\overline{AB}\parallel\overline{CD}$

a different way...?

$\overline{AB}$ and $\overline{CD}$ are congruent and parallel so ABCD is a parallelogram by Theorem 8.9