8.1 Find Angle Measures in Polygons

consecutive vertices – two vertices that are endpoints of the same side

diagonal – a segment that joins two nonconsecutive vertices

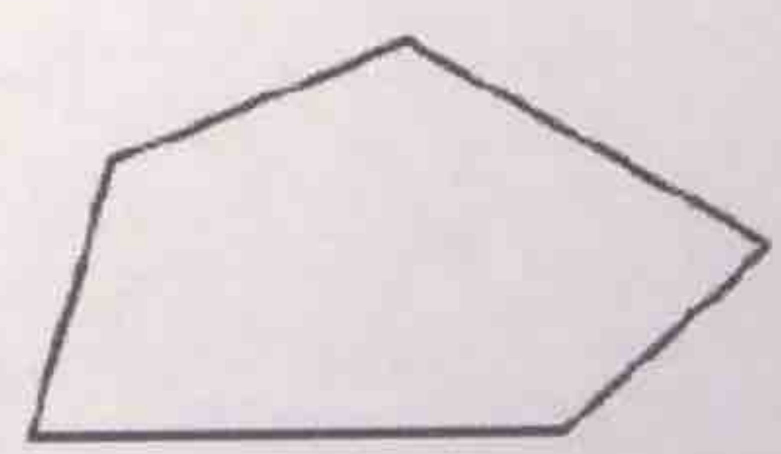
EXPLORE Find sums of interior angle measures

STEP 1 ***Draw polygons*** Use a straightedge to draw convex polygons with three sides, four sides, five sides, and six sides. An example is shown.

STEP 2 ***Draw diagonals*** In each polygon, draw all the diagonals from one vertex. A *diagonal* is a segment that joins two nonconsecutive vertices. Notice that the diagonals divide the polygon into triangles.

STEP 3 ***Make a table*** Copy the table below. By the Triangle Sum Theorem, the sum of the measures of the interior angles of a triangle is 180°. Use this theorem to complete the table.

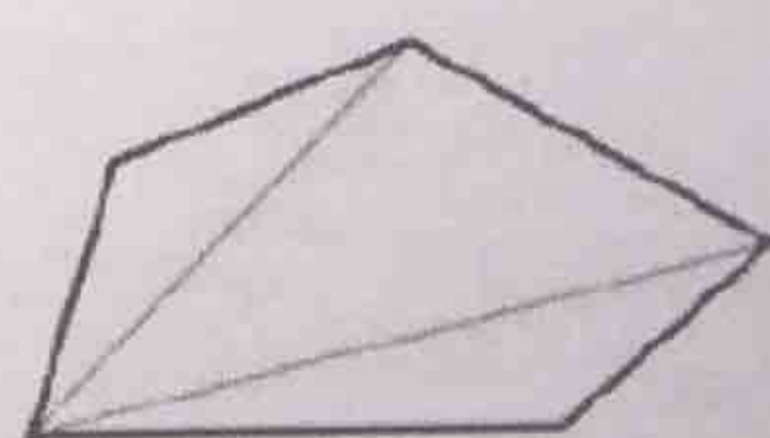
Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Triangle	3	1	$1 \cdot 180° = 180°$
Quadrilateral	4	2	$2 \cdot 180° = 360°$
Pentagon	5	3	$3 \cdot 180° = 540°$
Hexagon	6	4	$4 \cdot 180° = 720°$



Look for a pattern in the last column of the table...

What is the sum of the interior angles for a convex heptagon?

$\boxed{900°}$



What is the sum of the interior angles for a convex octagon?

$\boxed{1080°}$

What is the sum of the interior angles for a convex n-gon?

$\boxed{(n-2) \cdot 180°}$

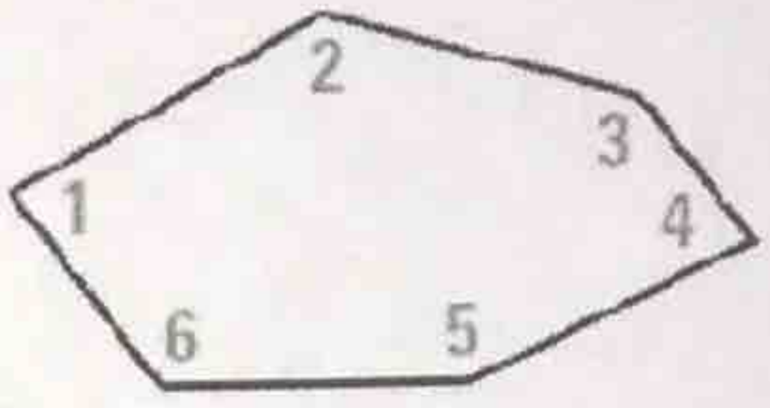
THEOREMS — *For Your Notebook*

THEOREM 8.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n-2) \cdot 180°$.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n-2) \cdot 180°$

Proof: Ex. 33, p. 512 (for pentagons)



$n = 6$

COROLLARY TO THEOREM 8.1 Interior Angles of a Quadrilateral

The sum of the measures of the interior angles of a quadrilateral is 360°.

Proof: Ex. 34, p. 512

Ex 1: Find the sum of the measures of the interior angles of a convex decagon.

$(n-2) \cdot 180°$

$(10-2) \cdot 180°$

$(8) \cdot 180°$

$\boxed{1440°}$

Ex 2: The sum of the measures of the interior angles of a convex polygon is 2340°. Classify the polygon by the number of sides.

$(n-2) \cdot 180° = 2340°$

$n-2 = 13$

$n = 15$

$\boxed{15\text{-gon}}$

Ex 3: A convex pentagon has the following interior angle measures: 110°, 92°, 84°, 100°, and x°. What is the value of x?

pentagon:

$(n-2) \cdot 180°$

$(5-2) \cdot 180°$

$(3) \cdot 180°$

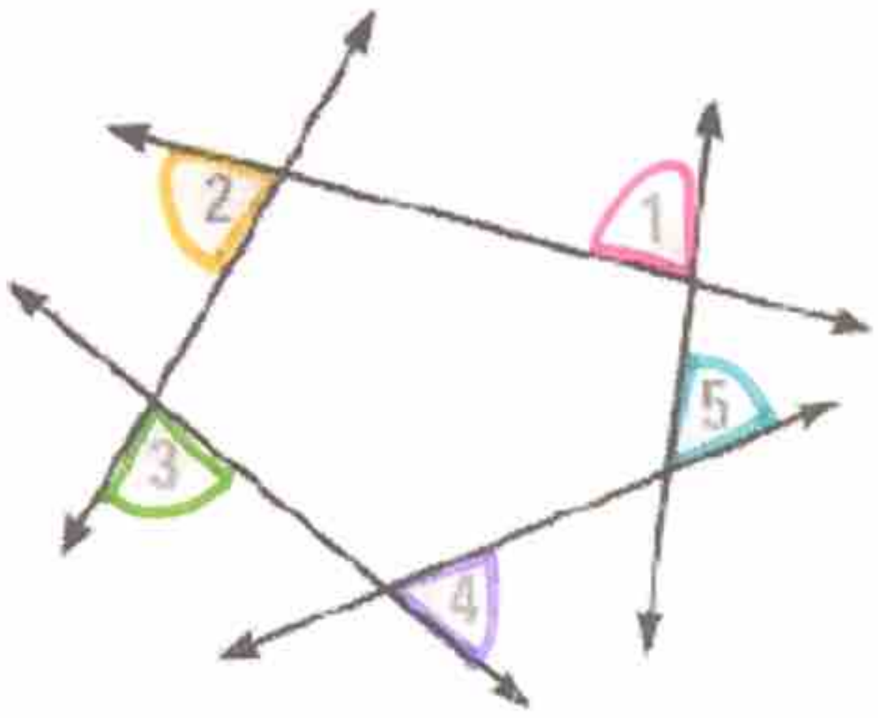
$540°$

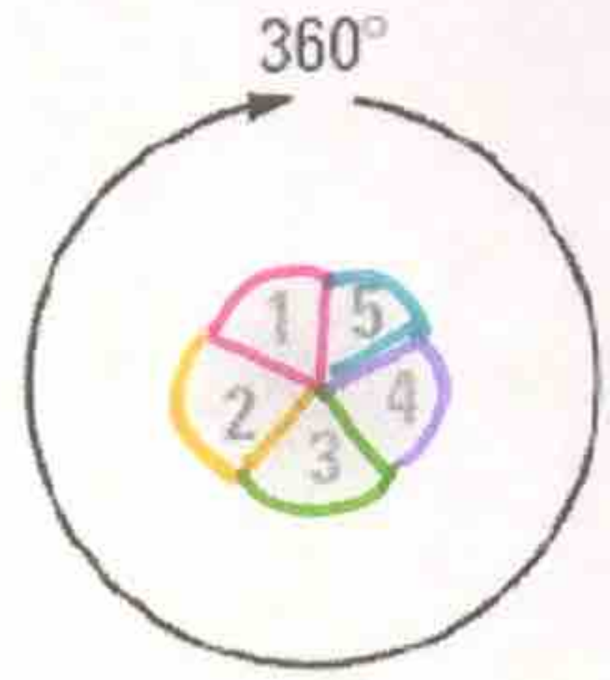
$110° + 92° + 84° + 100° + x° = 540°$

$x + 386 = 540$

$\boxed{x = 154}$

In general, this sum is 360° for any convex polygon.





STEP 1 **Shade** one exterior angle at each vertex.

STEP 2 **Cut** out the exterior angles.

STEP 3 **Arrange** the exterior angles to form 360°.

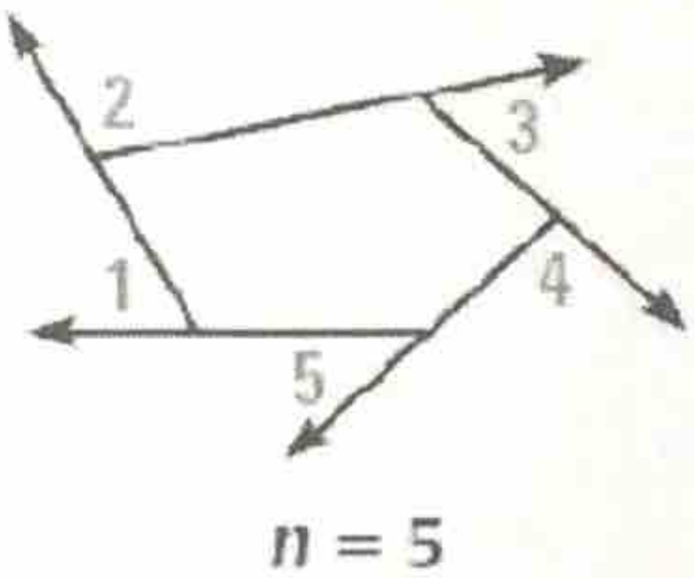
THEOREM *For Your Notebook*

THEOREM 8.2 Polygon Exterior Angles Theorem

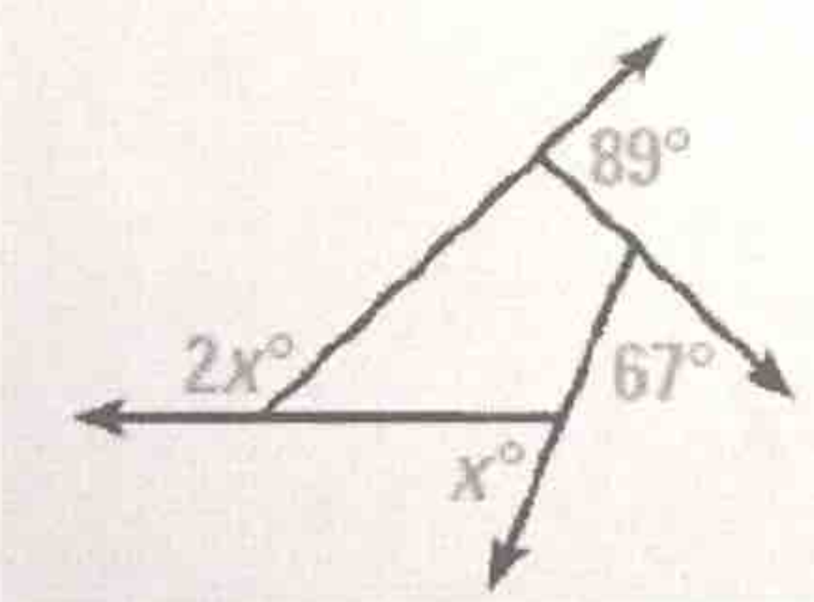
The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$

Proof: Ex. 35, p. 512



Ex 4: What is the value of x in the diagram?



$2x + 89 + 67 + x = 360$

$3x + 156 = 360$

$3x = 204$

$\boxed{x = 68}$

Ex 5: A stop sign is shaped like a regular octagon. Find the measure of each interior angle and the measure of each exterior angle.

interior ∠s:

$(n-2)\cdot 180°$

$(8-2)\cdot 180°$

$(6)\cdot 180°$

$1080°$

$8\angle s = 1080°$

$\boxed{1\angle = 135°}$

exterior ∠s:

$8\angle s = 360°$

$\boxed{1\angle = 45°}$