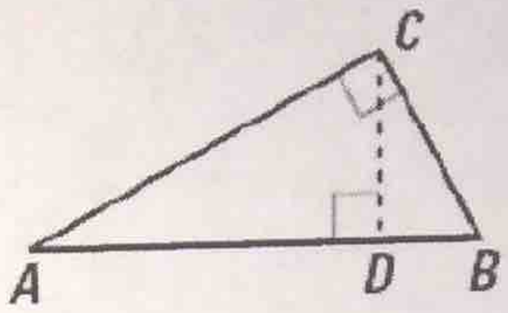
7.3 Use Similar Right Triangles

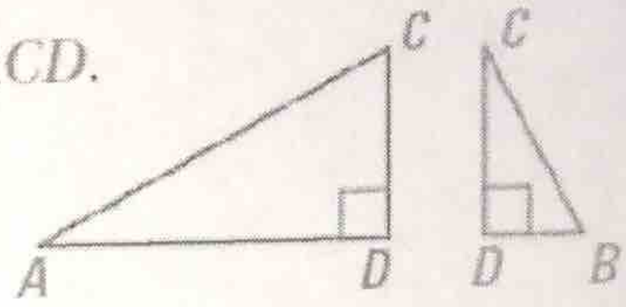
THEOREM — *For Your Notebook*

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



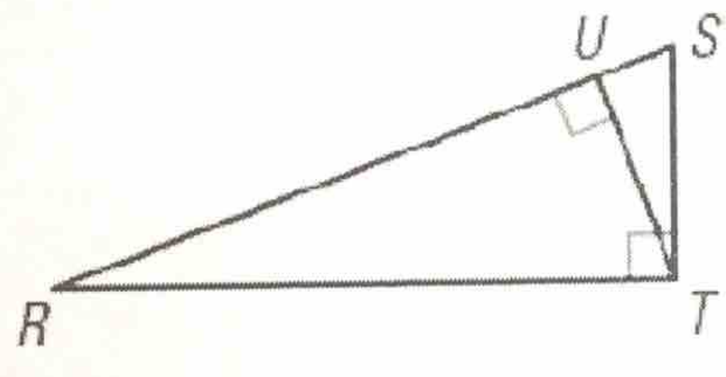
$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

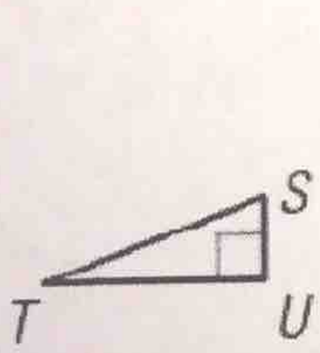


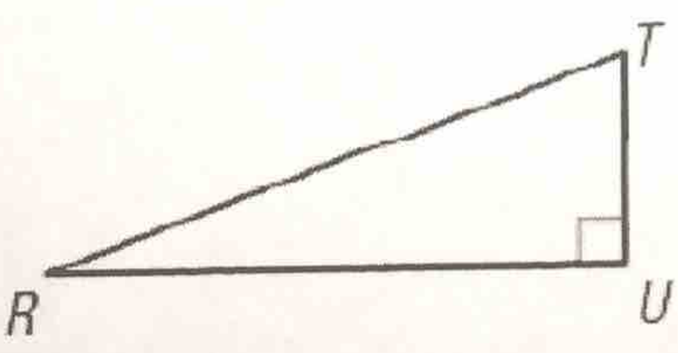
Proof: below; Ex. 35, p. 456

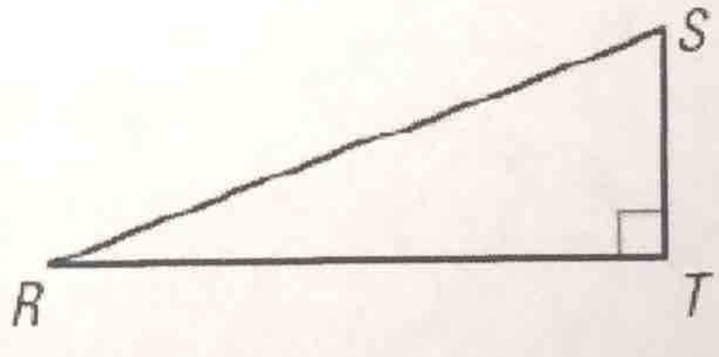
Identify the similar triangles in the diagram.

Ex 1:



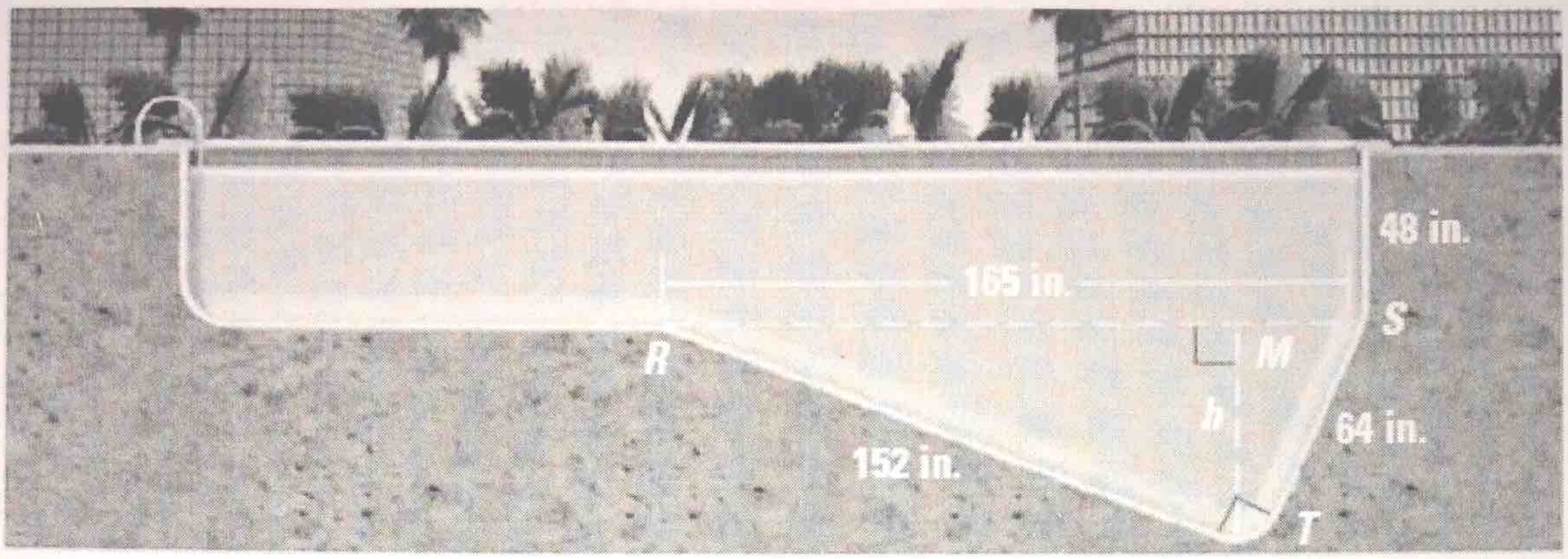


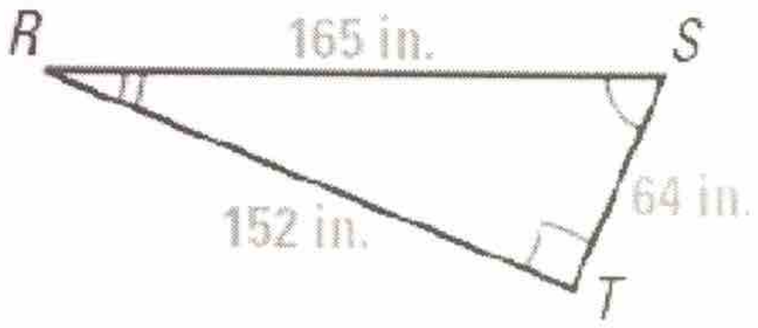


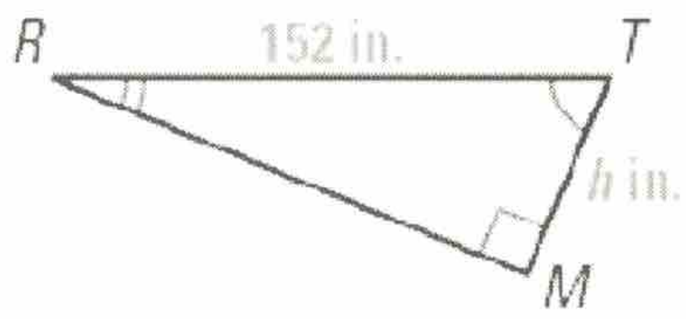


$\triangle TSU \sim \triangle RTU \sim \triangle RST$

Ex 2: The diagram shows a cross-section of a swimming pool. What is the maximum depth of the pool?







$\Delta RST \sim \Delta RTM \sim \Delta TSM$

Since $\Delta RST \sim \Delta RTM$

$$\frac{TM}{ST} = \frac{TR}{SR}$$

$$\frac{h}{64} = \frac{152}{165}$$

$$165h = 64(152)$$

$$165h = 9728$$

$$h \approx 59 \text{ in}$$

$$\text{Total height} \approx 48 + h$$
$$\approx 48 + 59$$
$$\boxed{\approx 107 \text{ in}}$$

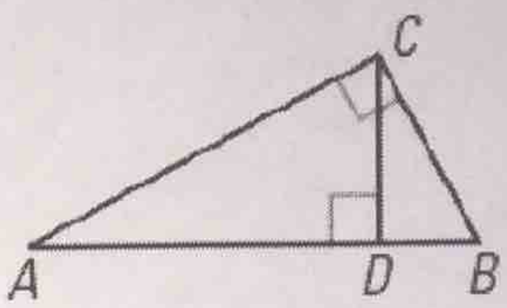
THEOREMS — For Your Notebook

THEOREM 7.6 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

Proof: Ex. 36, p. 456



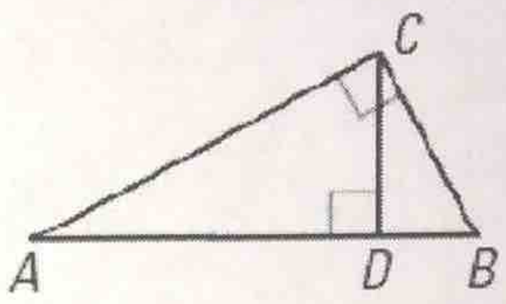
$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 7.7 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

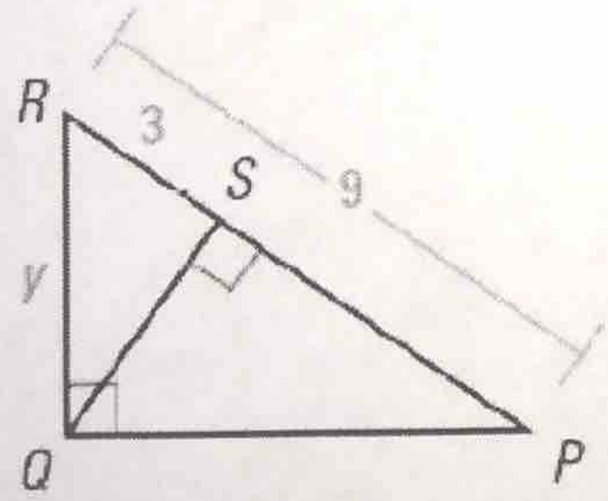
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

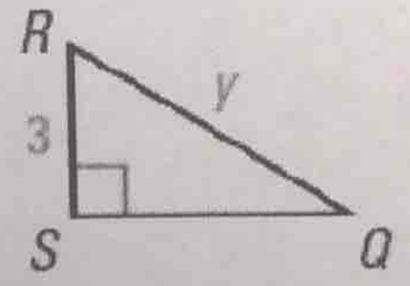
Proof: Ex. 37, p. 456

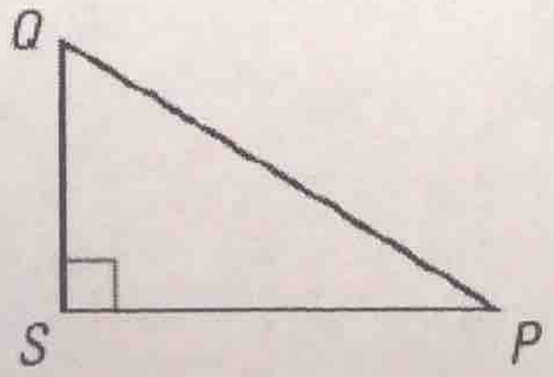


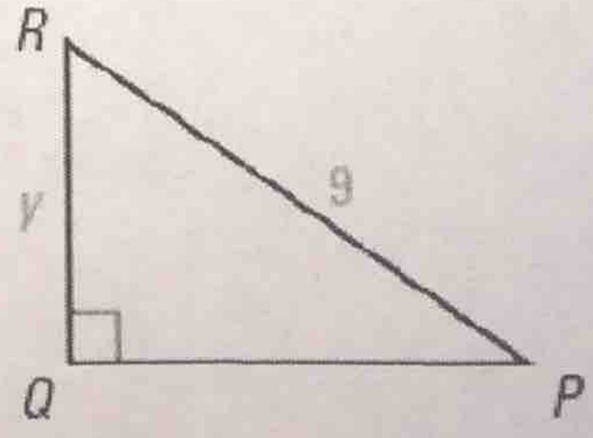
$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

Ex 3: Find the value of y. Write the answer in simplest radical form.









$\frac{9}{y} = \frac{y}{3}$

$y^2 = 27$

$y = 3\sqrt{3}$ units

Ex 4: Approximate the height of the gym wall.

By Theorem 7.6, $\frac{w}{8.5} = \frac{8.5}{5}$

$5w = 72.25$

$w = 14.45$

Total height $= 5 + w$

$= 5 + 14.45$

$= 19.45$

≈ 19.5 ft