Need graph paper

6.7 Perform Similarity Transformations

dilation – a transformation that stretches or shrinks a figure to create a similar figure

center of dilation – a fixed point with respect to which a figure is enlarged or reduced

scale factor of a dilation – the ratio of a side length of the image to the corresponding side length of the original figure $\dfrac{image}{original}$

KEY CONCEPT   *For Your Notebook*

Coordinate Notation for a Dilation

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a **reduction**. If $k > 1$, the dilation is an **enlargement**.

<u>Ex 1</u>: Draw a dilation of quadrilateral ABCD with vertices A(2, 1), B(4, 1), C(4, -1), and D(1, -1). Use a scale factor of 2.

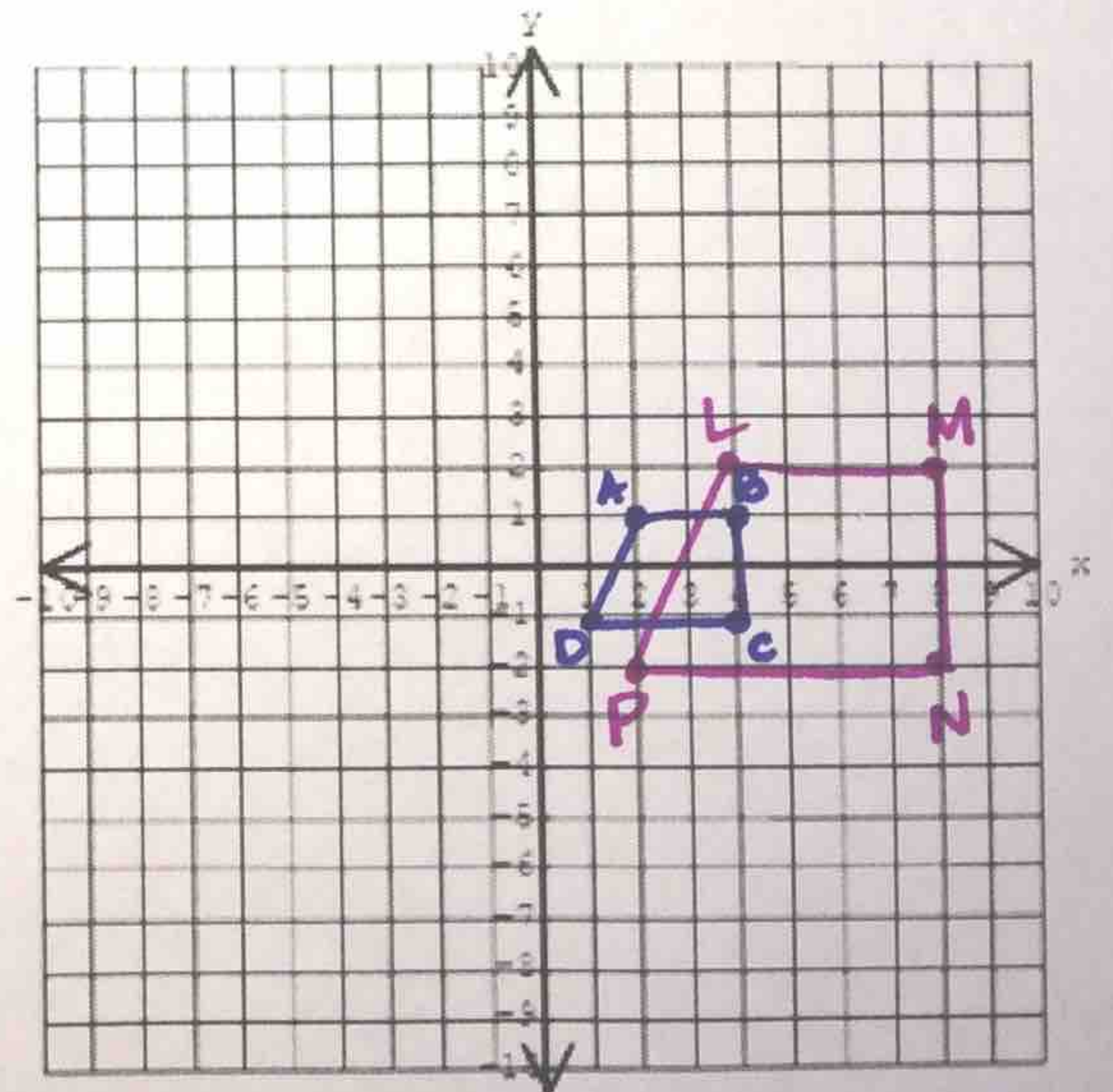$(x,y) \rightarrow (2x,2y)$

$A(2,1) \rightarrow L(4,2)$

$B(4,1) \rightarrow M(8,2)$

$C(4,-1) \rightarrow N(8,-2)$

$D(1,-1) \rightarrow P(2,-2)$

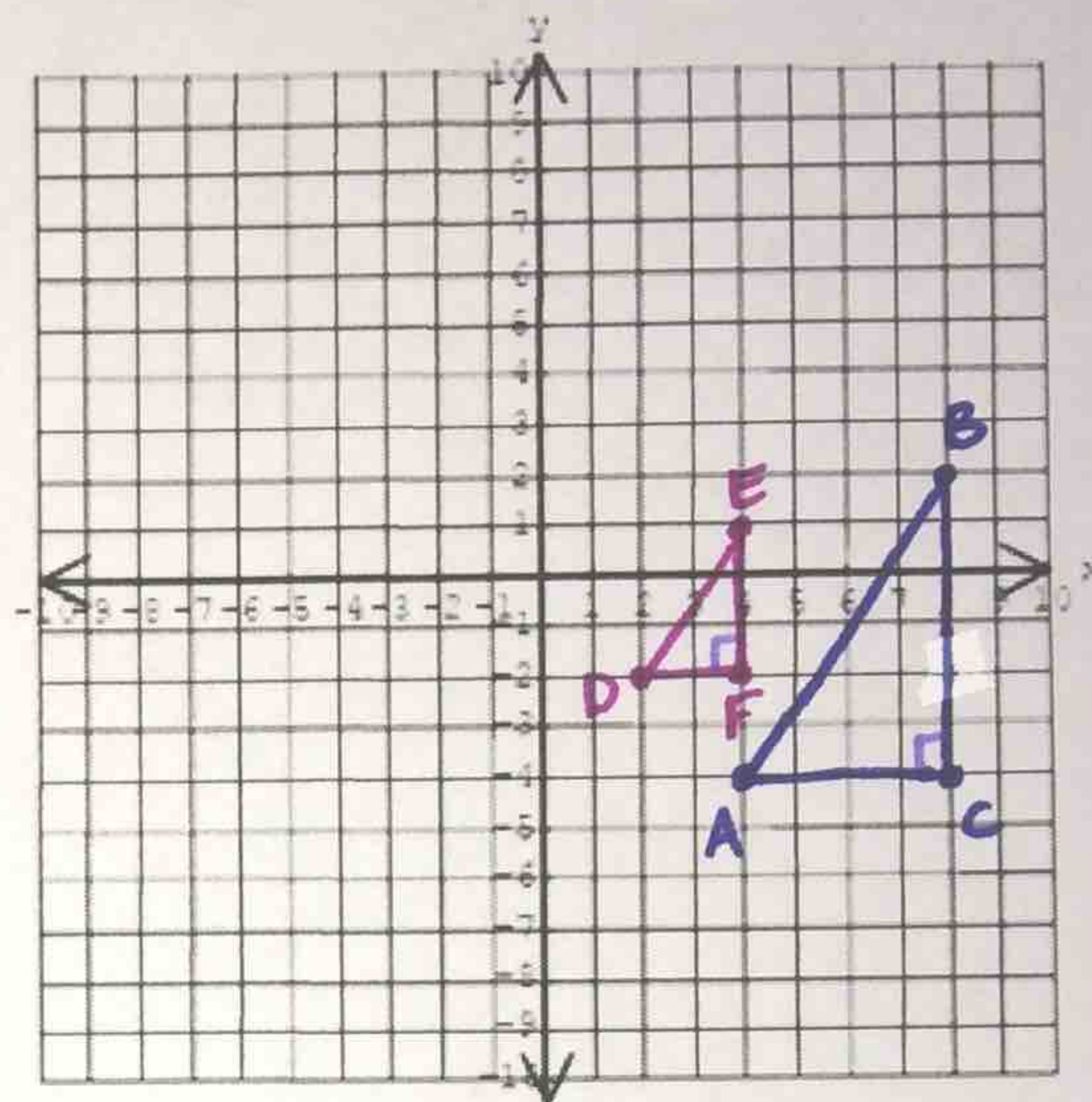Ex 2: A triangle has the vertices A(4, -4), B(8, 2), and C(8, -4). The image of △ ABC after a dilation with a scale factor of ½ is △ DEF. Sketch both triangles and verify that they are similar.

$$(x,y) \rightarrow (\tfrac{1}{2}x, \tfrac{1}{2}y)$$

$$A(4,-4) \rightarrow D(2,-2)$$

$$B(8,2) \rightarrow E(4,1)$$

$$C(8,-4) \rightarrow F(4,-2)$$

$$\angle C \cong \angle F$$

$$\frac{AC}{DF} \overset{?}{=} \frac{BC}{EF}$$

$$\frac{4}{2} \overset{?}{=} \frac{6}{3}$$

$$2 = 2 \checkmark$$

So △ABC ~ △DEF by SAS Similarity Theorem

Ex 3: You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of the reduction?

$$\text{scale factor} = \frac{\text{image}}{\text{original}} = \frac{1.1 \text{ in}}{4 \text{ in}}$$

$$\boxed{\text{scale factor} = \frac{11}{40}}$$

Generally, for a center of dilation at the origin, a point of the figure and its image lie on the same ray from the origin. If a point is the origin, its image is also the origin.

Ex 4: You want to create a quadrilateral EFGH that is similar to quadrilateral PQRS. What are the coordinates of H?

$$(x,y) \rightarrow (kx, ky)$$

$P(3,0) \rightarrow E(9,0)$ $k=3$

$Q(1,1) \rightarrow F(3,3)$ $k=3$

$R(0,2) \rightarrow G(0,6)$ $k=3$

So the image is a dilation with a scale factor of 3.

$$(x,y) \rightarrow (3x, 3y)$$

$$S(4,5) \rightarrow H(3\cdot4, 3\cdot5)$$

$$= \boxed{H(12,15)}$$