

6.5 Prove Triangles Similar by SSS and SAS

THEOREM

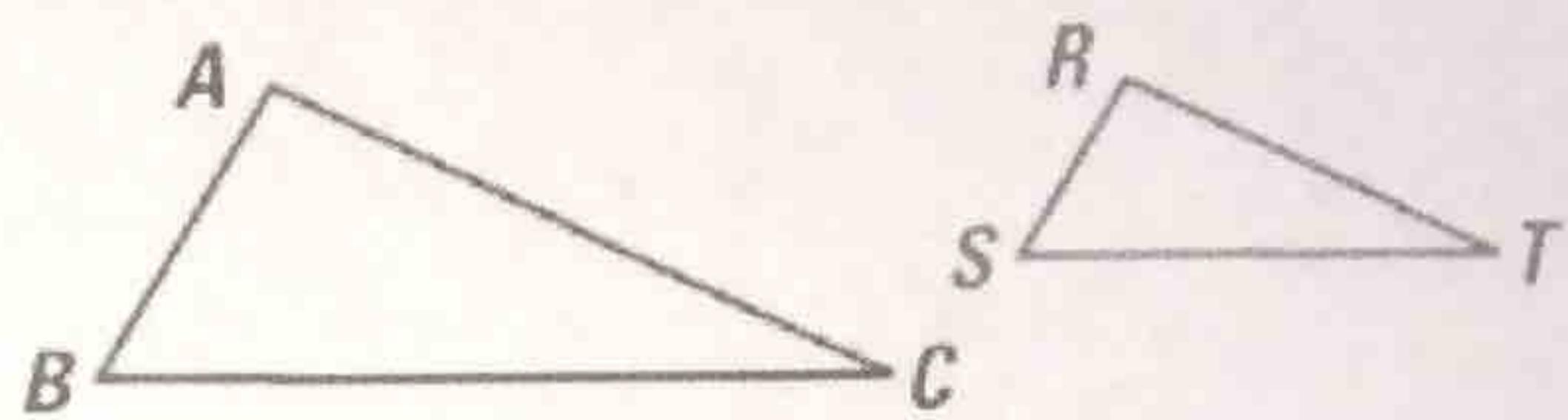
For Your Notebook

THEOREM 6.2 Side-Side-Side (SSS) Similarity Theorem

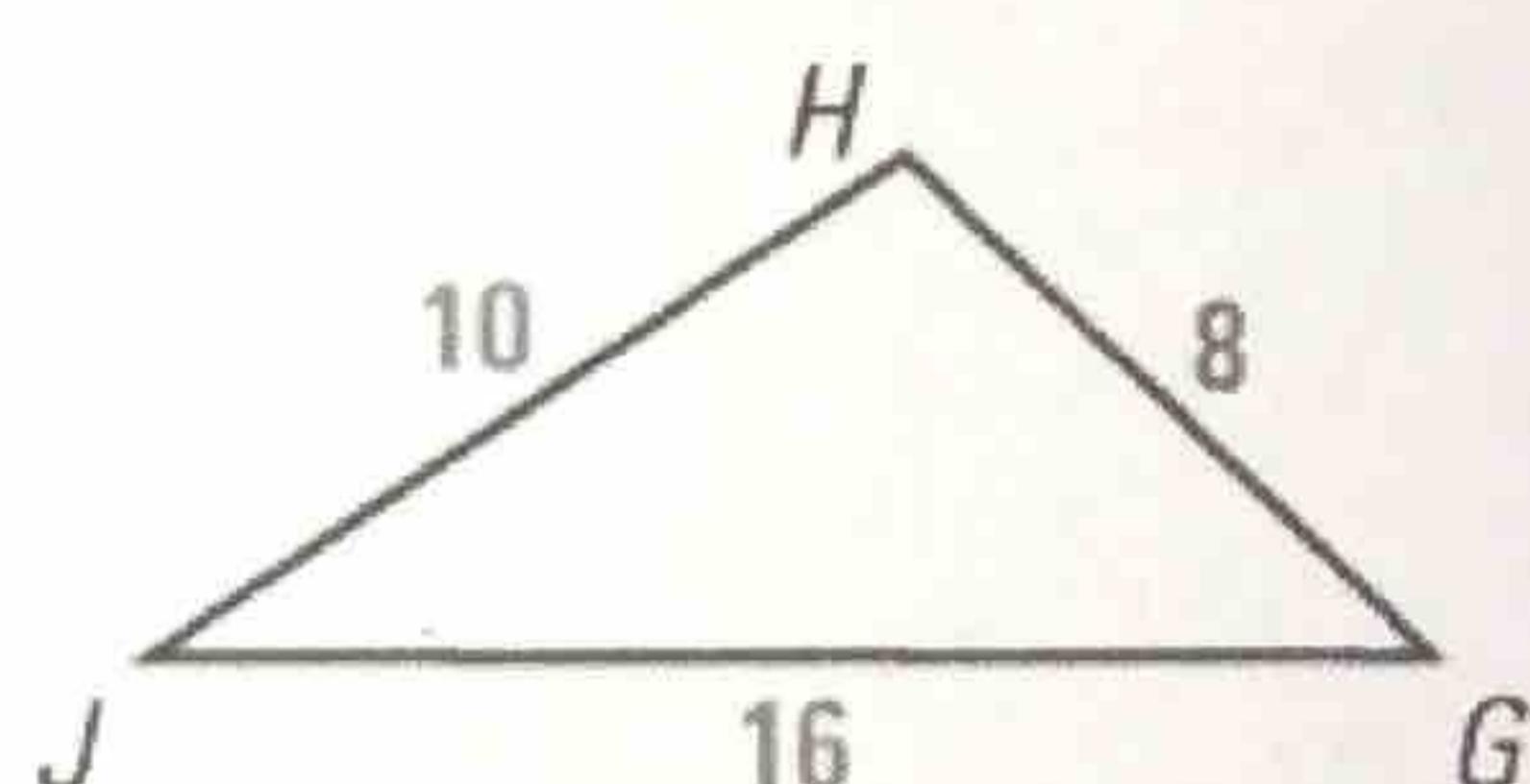
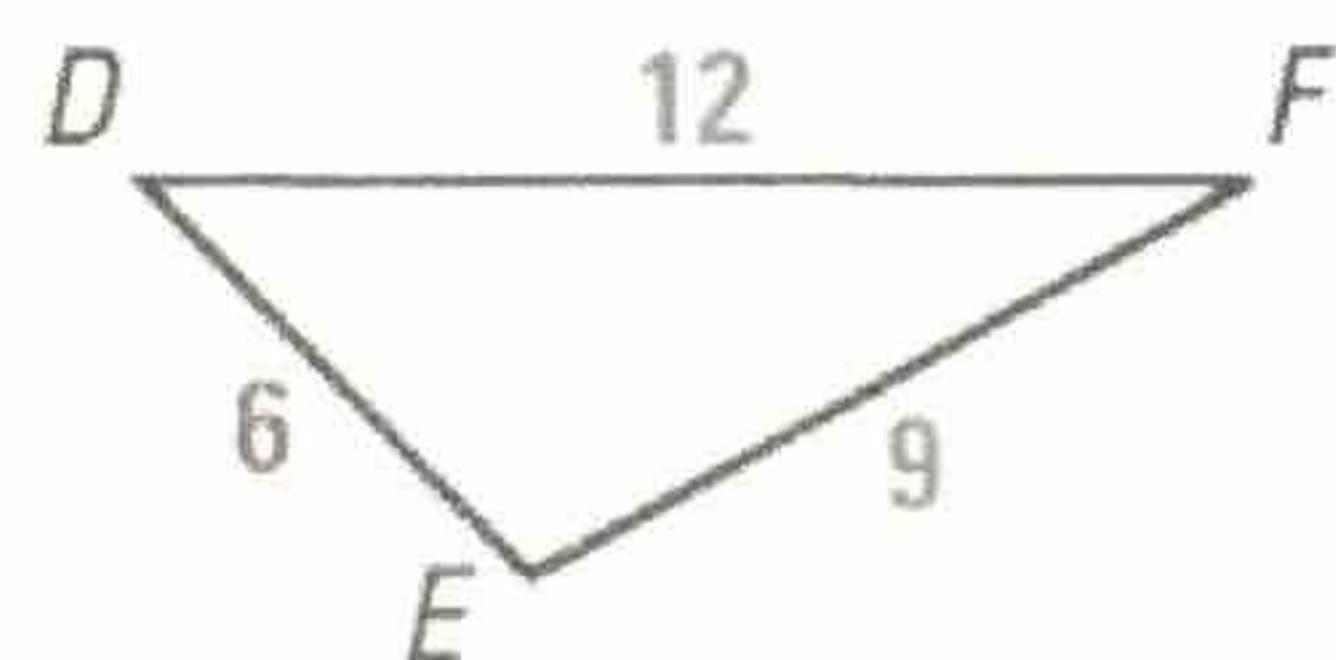
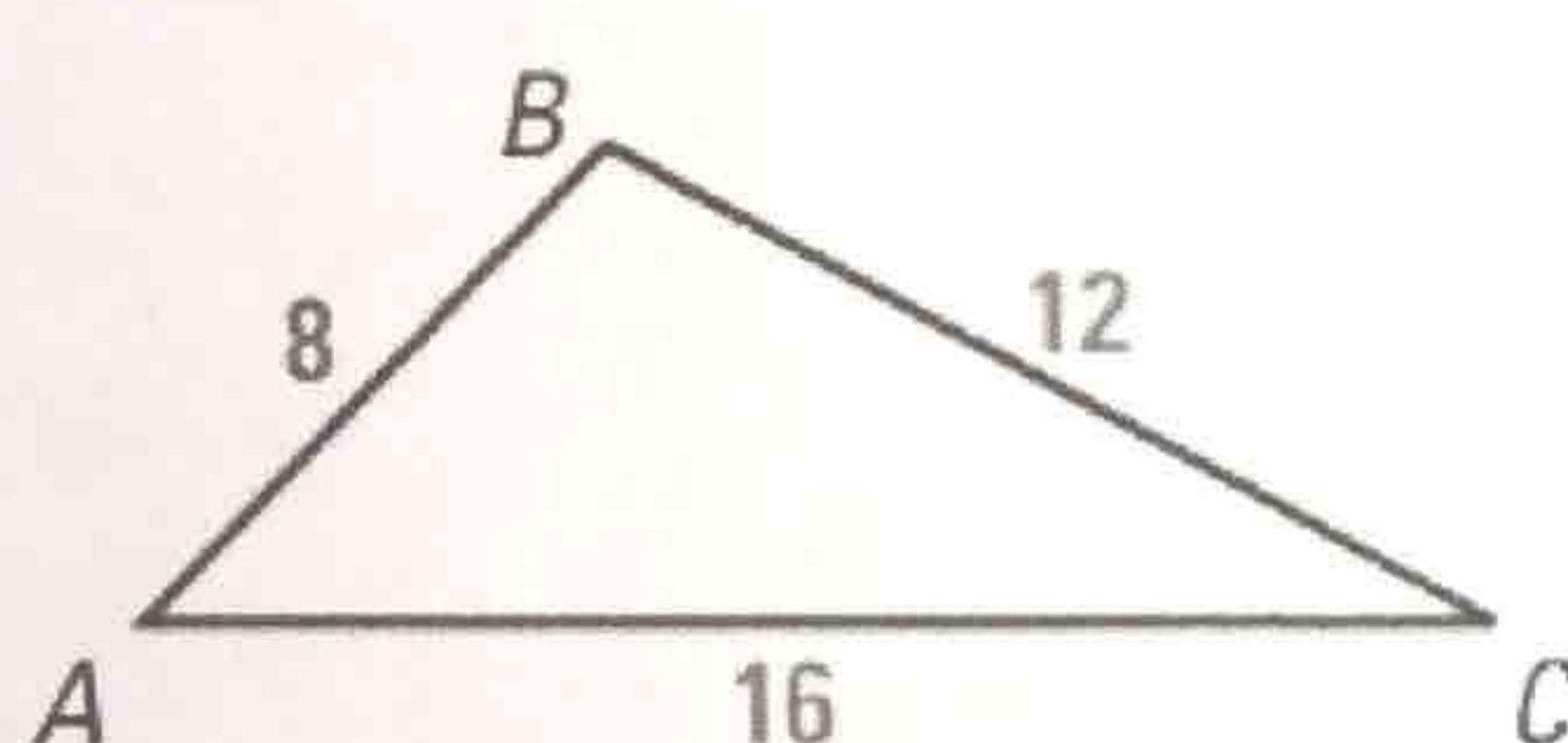
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof: p. 389



Ex 1: Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Compare $\triangle ABC$ and $\triangle DEF$:

shortest sides:

$$\frac{AB}{DE} = \frac{8}{6} = \frac{4}{3}$$

longest sides:

$$\frac{CA}{FD} = \frac{16}{12} = \frac{4}{3}$$

remaining sides:

$$\frac{BC}{EF} = \frac{12}{9} = \frac{4}{3}$$

so

$\boxed{\triangle ABC \sim \triangle DEF}$

Compare $\triangle ABC$ and $\triangle GHJ$

shortest sides:

$$\frac{AB}{GH} = \frac{8}{8} = 1$$

longest sides:

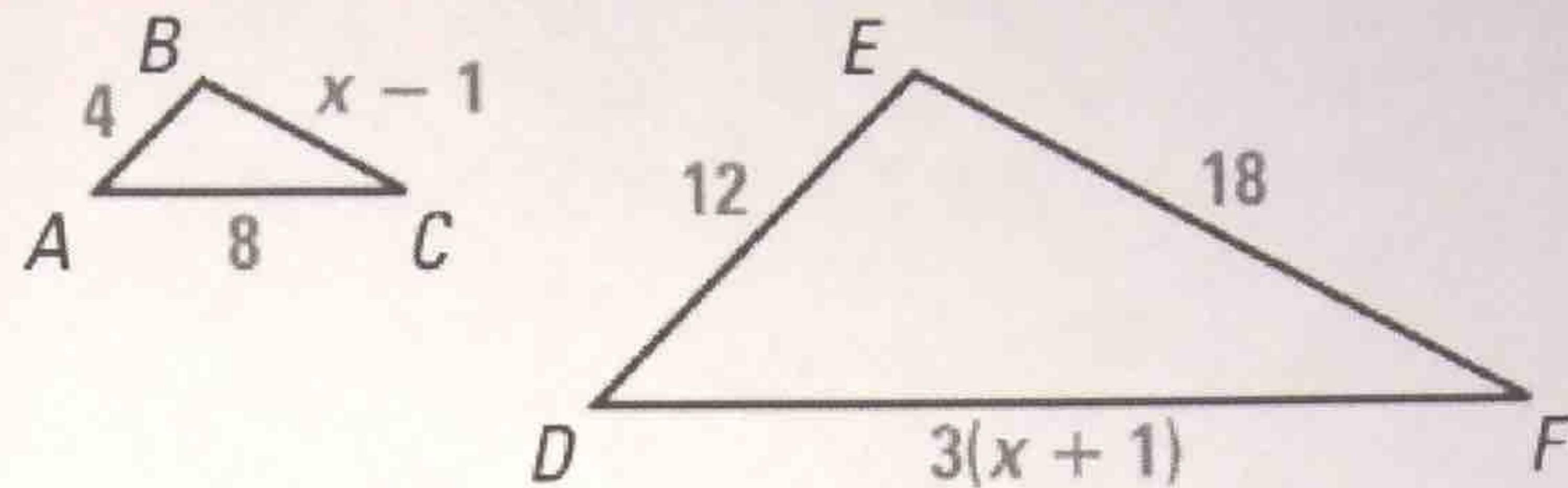
$$\frac{CA}{JG} = \frac{16}{16} = 1$$

remaining sides:

$$\frac{BC}{HJ} = \frac{12}{10} = \frac{6}{5}$$

$\boxed{\triangle ABC \text{ and } \triangle GHJ \text{ are not similar}}$

Ex 2: Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{4}{12} = \frac{x-1}{18}$$

$$\frac{1}{3} = \frac{x-1}{18}$$

$$18 = 3(x-1)$$

$$18 = 3x - 3$$

$$21 = 3x$$

$$\boxed{x=7}$$

Check:

$$BC = x-1$$

$$= 7-1$$

$$= 6$$

$$DF = 3(x+1)$$

$$= 3(7+1)$$

$$= 3(8)$$

$$= 24$$

$$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF}$$

$$\frac{4}{12} \stackrel{?}{=} \frac{6}{18}$$

$$\frac{1}{3} \stackrel{?}{=} \frac{1}{3}$$

$$\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF}$$

$$\frac{4}{12} \stackrel{?}{=} \frac{8}{24}$$

$$\frac{1}{3} \stackrel{\checkmark}{=} \frac{1}{3}$$

THEOREM

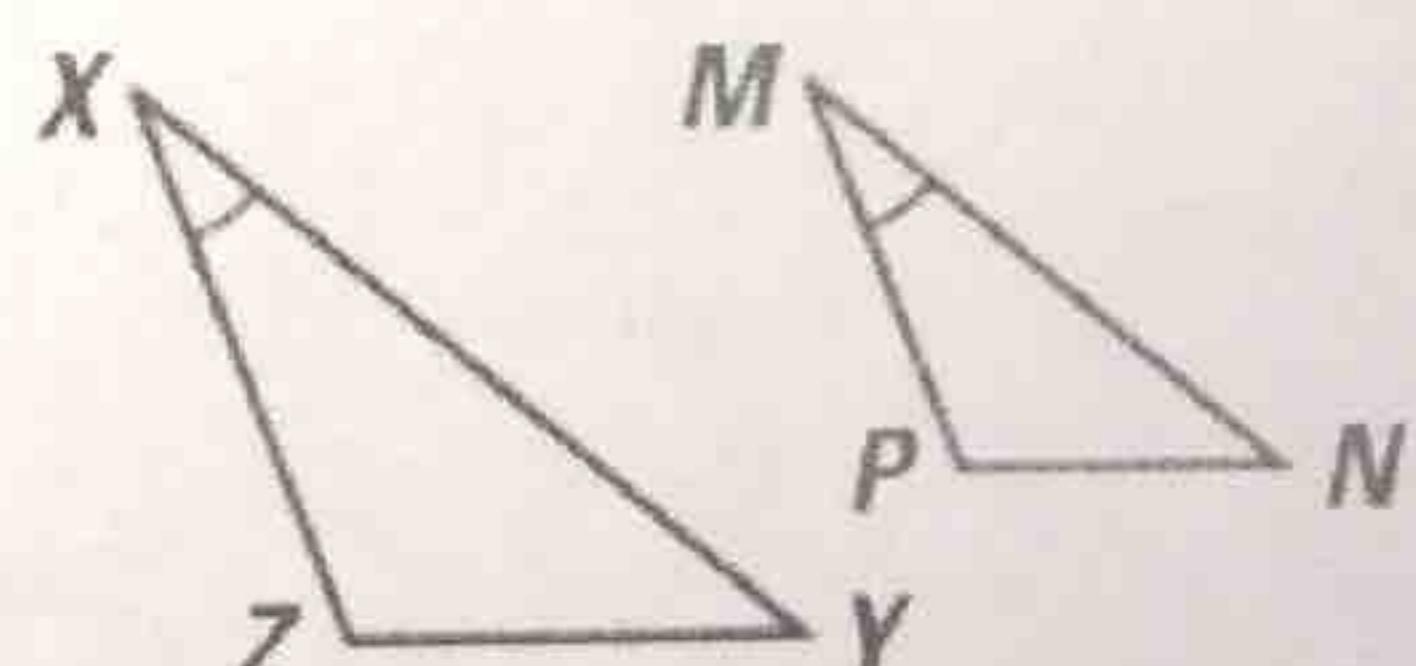
For Your Notebook

THEOREM 6.3 Side-Angle-Side (SAS) Similarity Theorem

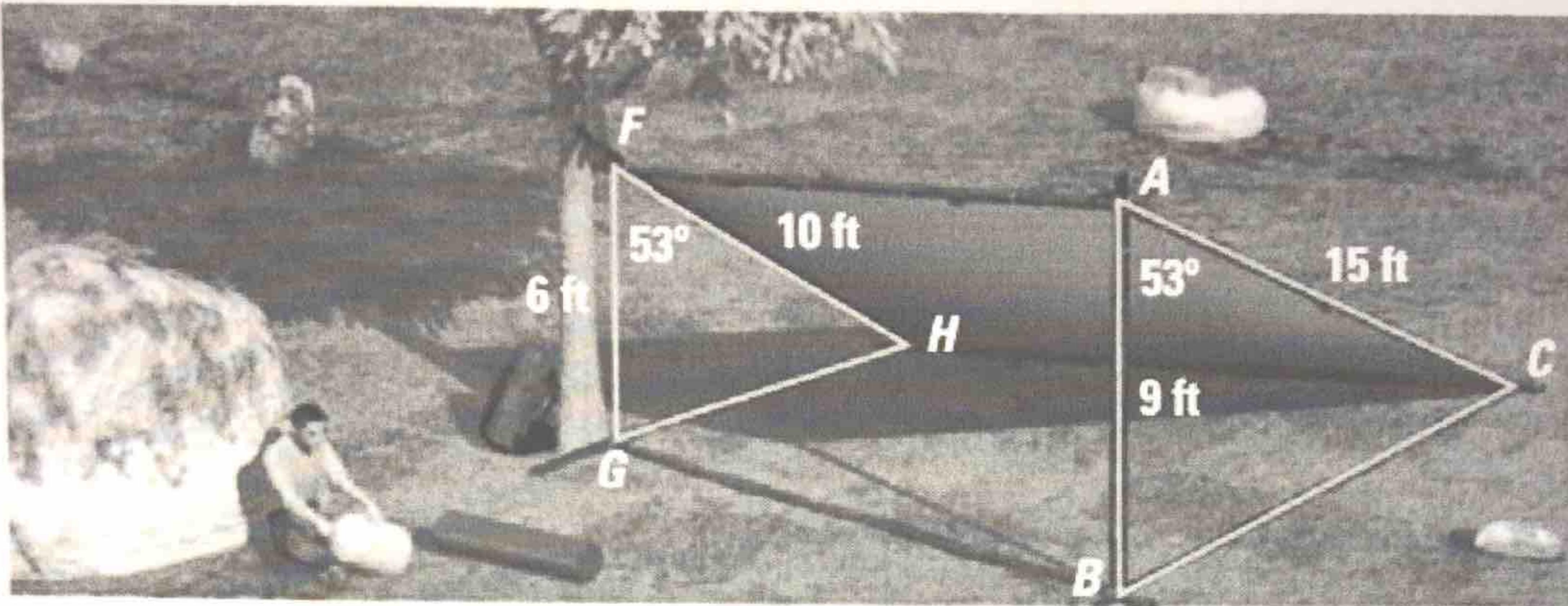
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof: Ex. 37, p. 395



Ex 3: You are building a shelter starting from a tree branch. Is it possible to construct the right end so it is similar to the left end using the angle measure and lengths shown?



$$\angle A \cong \angle F$$

shorter sides: $\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$

longer sides: $\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$

So $\triangle ABC \sim \triangle FGH$ by the SAS Similarity Theorem.

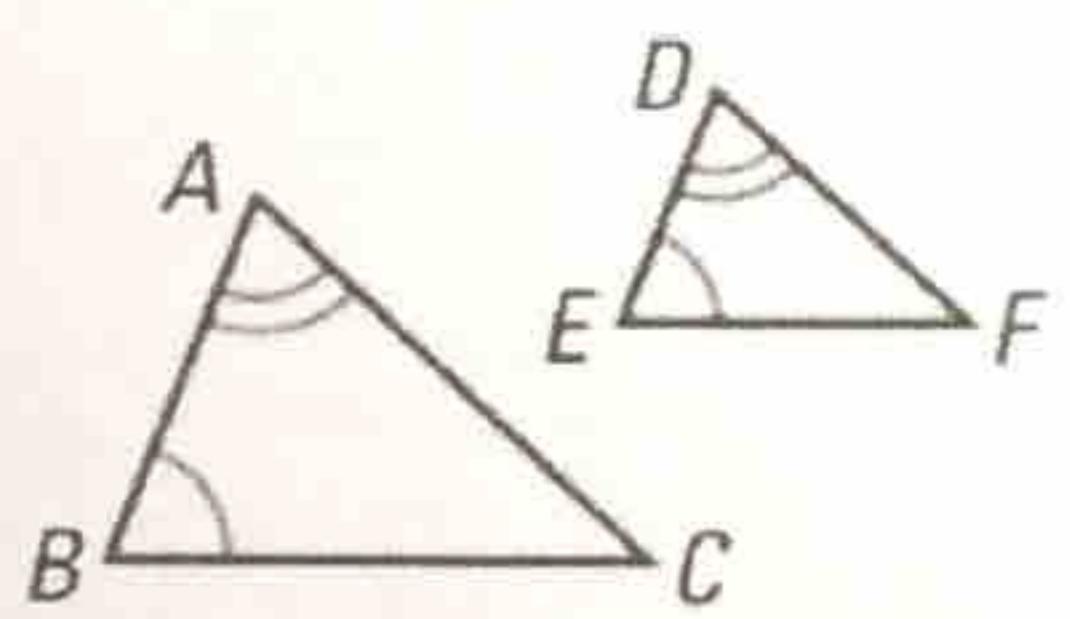
Yes, you can make the right end similar to the left end.

CONCEPT SUMMARY

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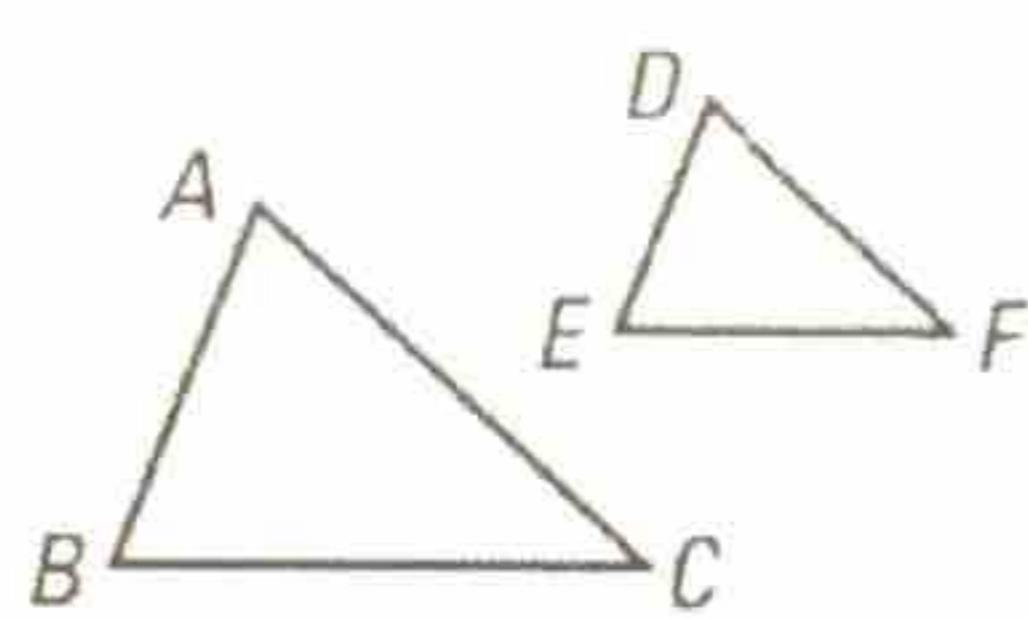
Triangle Similarity Postulate and Theorems

AA Similarity Postulate



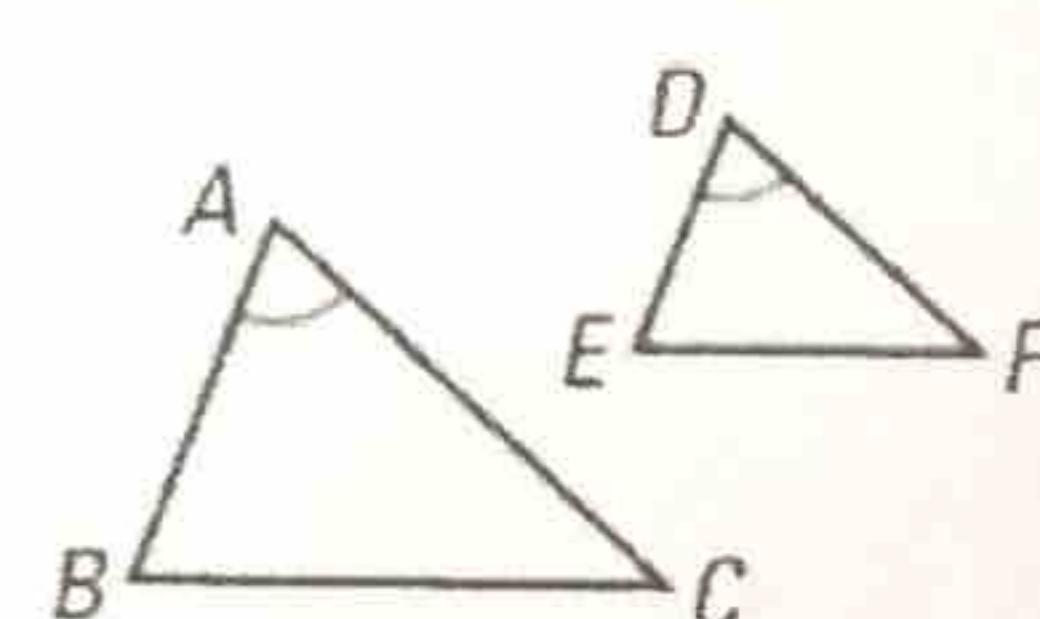
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem



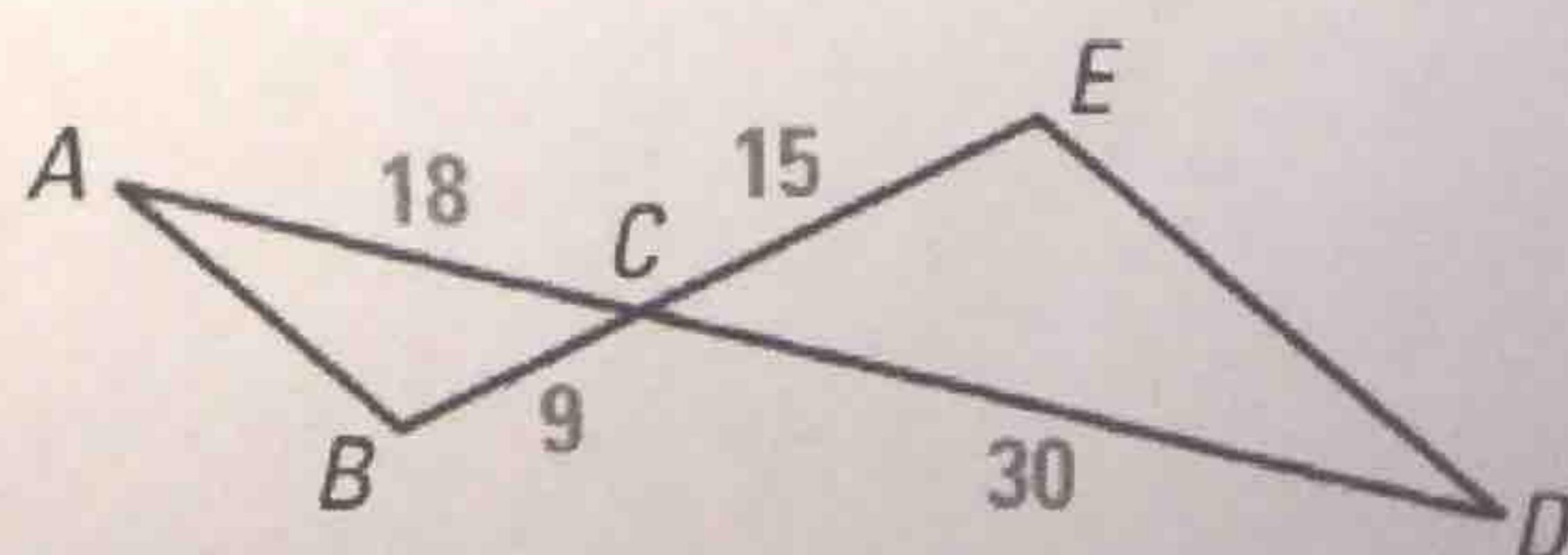
If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Ex 4: Tell what method you would use to show that the triangles are similar.

shorter sides: $\frac{BC}{EC} = \frac{9}{15} = \frac{3}{5}$

longer sides: $\frac{CA}{CD} = \frac{18}{30} = \frac{3}{5}$

$\angle ACB \cong \angle DCE$ (Vertical \angle s)



$\triangle ACB \sim \triangle DCE$ by SAS Similarity Theorem