6.1 Ratios, Proportions, and the Geometric Mean

ratio of a to b – a comparison of a and b, which are 2 numbers or quantities and $b \neq 0$; can be written in one of 3 ways: a:b, a to b, $\frac{a}{b}$

Simplify each ratio.

Ex 1: 12 km : 3 km

$\frac{12\,km}{3\,km} = \frac{12 \div 3}{3 \div 3} = \boxed{\frac{4}{1}}$

4:1

4 to 1

Ex 2: 36 in : 9 ft

$\frac{36\,in}{9\,ft} \times \frac{1\,ft}{12\,in} = \frac{36}{108} \div \frac{36}{36} = \boxed{\frac{1}{3}}$

1:3

1 to 3

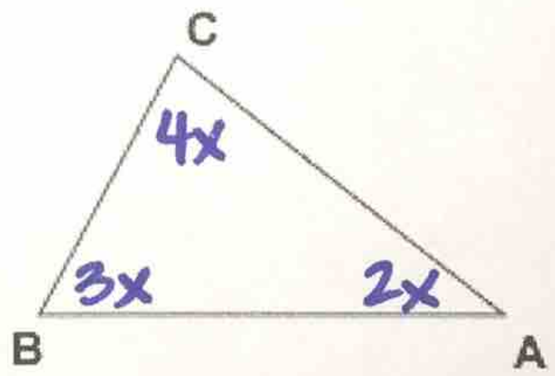
Useful Conversion Factors: 1 L = 1000 mL, 1 lb = 16 oz, 1 m = 100 cm

Ex 3: The measures of the angles of ΔABC are in the **extended ratio** 2:3:4. Find the measures of the angles.

$2x + 3x + 4x = 180$

$9x = 180$

$x = 20$



2x: $2(20)$ → 40°

3x: $3(20)$ → 60°

4x: $4(20)$ → 80°

proportion – an equation that states that 2 ratios are equal

means – numbers **b** and **c** in a proportion

extremes – numbers **a** and **d** in a proportion

extreme → $\frac{a}{b} = \frac{c}{d}$ ← mean

mean → $\frac{a}{b} = \frac{c}{d}$ ← extreme

KEY CONCEPT *For Your Notebook*

A Property of Proportions

1. **Cross Products Property** In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

$\frac{2}{3} = \frac{4}{6}$ $\quad 3 \cdot 4 = 12$, $\quad 2 \cdot 6 = 12$

Solve each proportion.

Ex 4: $\frac{8}{24} = \frac{x}{27}$

$8(27) = 24(x)$

$216 = 24x$

$\boxed{x = 9}$

Ex 5: $\frac{2}{x+3} = \frac{5}{4x}$

$2(4x) = 5(x+3)$

$8x = 5x + 15$

$3x = 15$

$\boxed{x = 5}$

KEY CONCEPT *For Your Notebook*

Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Ex 6: Find the geometric mean of 36 and 54.

$\frac{36}{x} = \frac{x}{54}$

$x^2 = (36)(54)$

$x^2 = 1944$

$x = \sqrt{324 \cdot 6}$

$\boxed{x = 18\sqrt{6}}$