Need protractor, compass, graph paper

5.4 Use Medians and Altitudes

median of a triangle - a segment from a vertex to the midpoint of the opposite side

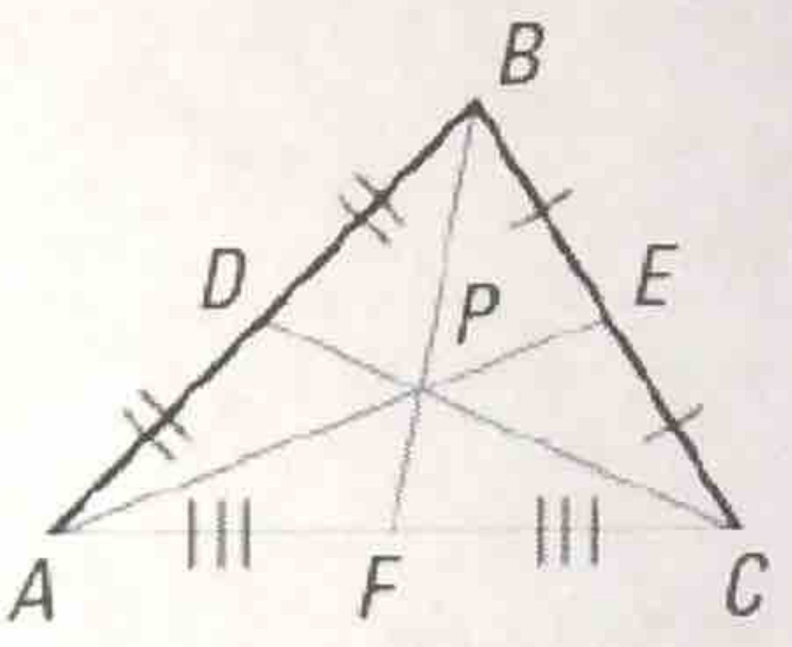
CENTROID - the point of concurrency of the 3 medians of a triangle, always inside the triangle, the point where a triangle will balance

THEOREM *For Your Notebook*

THEOREM 5.8 Concurrency of Medians of a Triangle

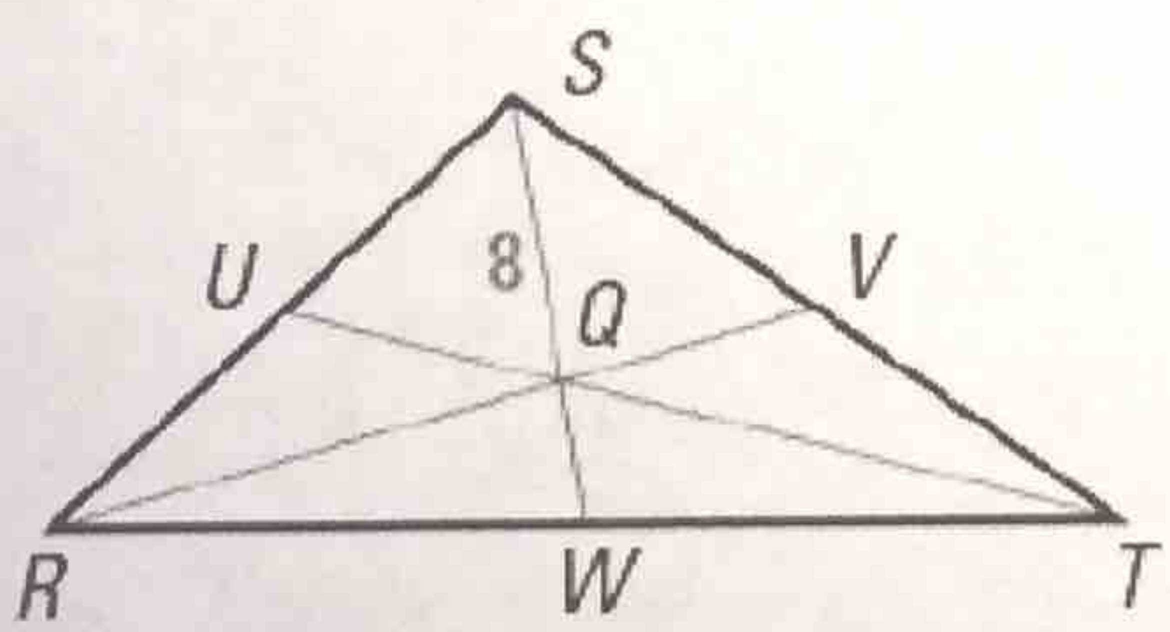
The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

The medians of $\triangle ABC$ meet at P and $AP = \frac{2}{3}AE$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CD$.



Proof: Ex. 32, p. 323; p. 934

Ex 1: In Δ RST, Q is the centroid and SQ = 8. Find QW and SW.



$SQ = \frac{2}{3} SW$

$8 = \frac{2}{3} SW$

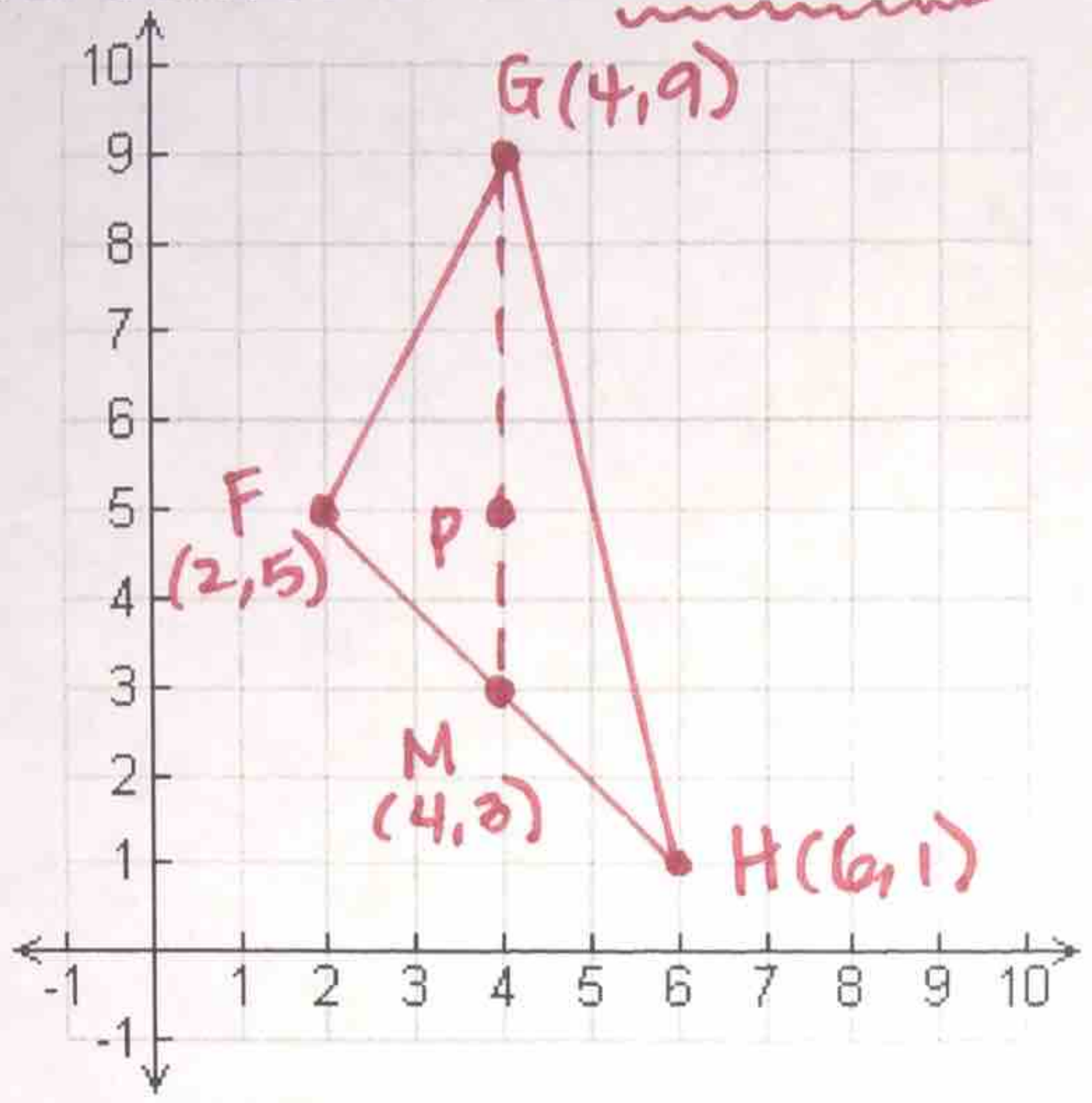
$\boxed{SW = 12}$

$SW = SQ + QW$

$12 = 8 + QW$

$\boxed{QW = 4}$

Ex 2: The vertices of Δ FGH are F(2, 5), G(4, 9), and H(6, 1). What are the coordinates of the centroid P of Δ FGH?



$M_{FH} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{2+6}{2}, \frac{5+1}{2}\right)$

$= (4, 3)$

GM = 6 units

$GP = \frac{2}{3} GM$

$GP = \frac{2}{3}(6)$

GP = 4 units (down from G)

$\boxed{P(4,5)}$

altitude of a triangle – the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side

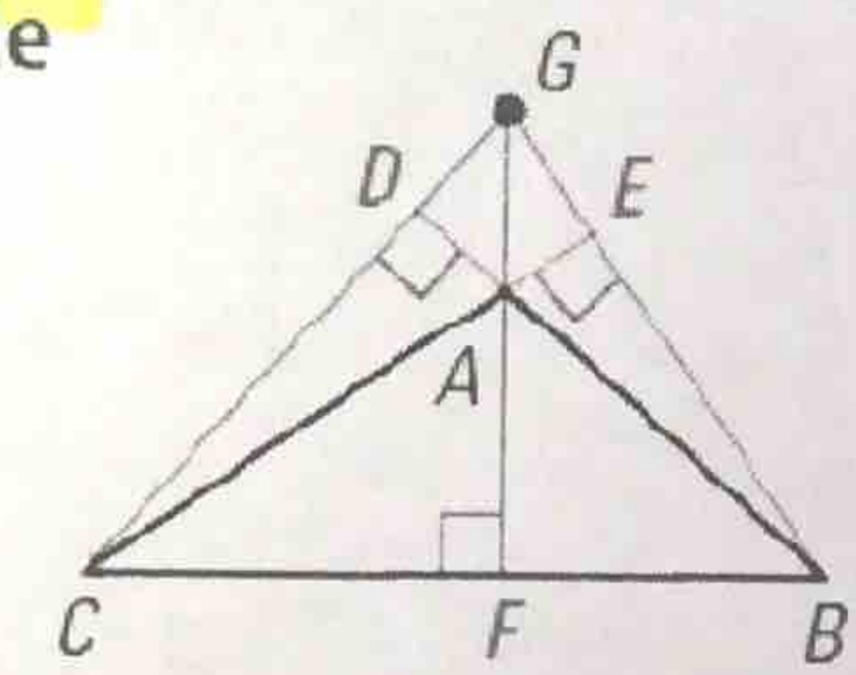
THEOREM *For Your Notebook*

THEOREM 5.9 Concurrency of Altitudes of a Triangle

The lines containing the altitudes of a triangle are concurrent.

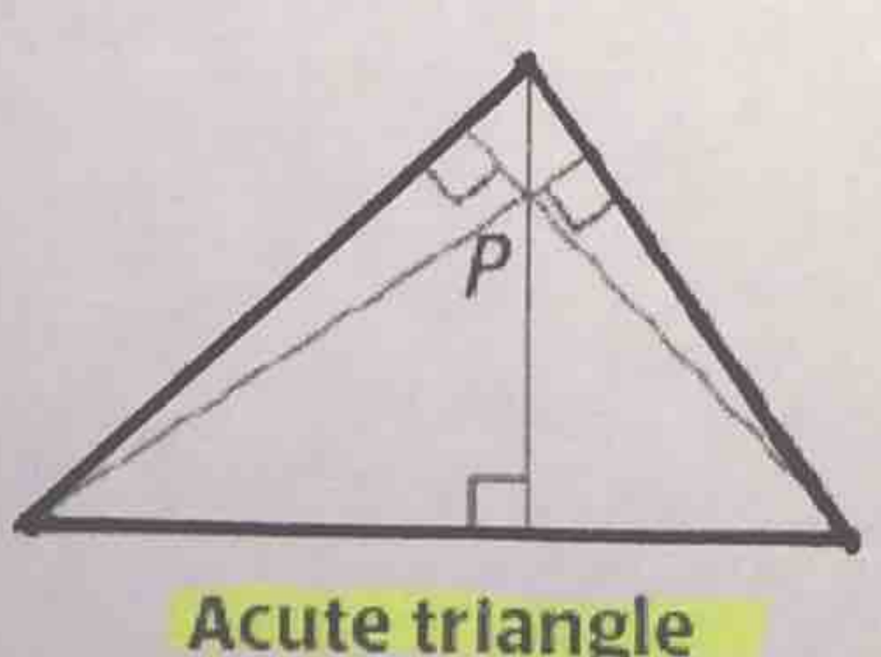
The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at G.

Proof: Exs. 29–31, p. 323; p. 936

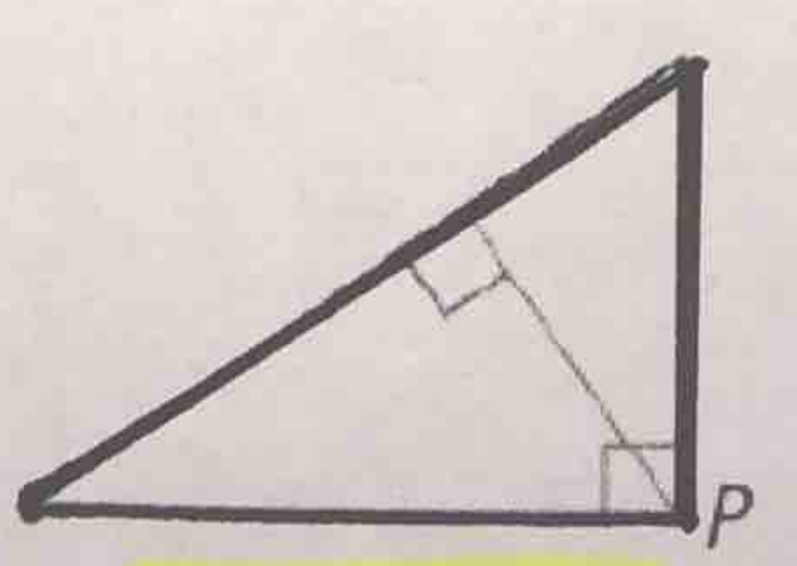


ORTHOCENTER – the point of concurrency of the 3 altitudes of a triangle

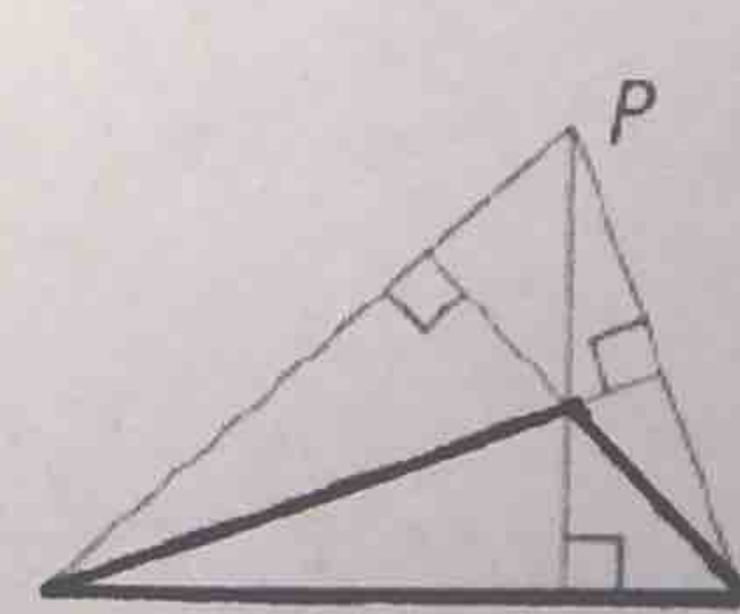
Ex 3: Construct the orthocenter P in an acute, a right, and an obtuse triangle. What do you notice about each orthocenter?



Acute triangle
P is inside triangle.



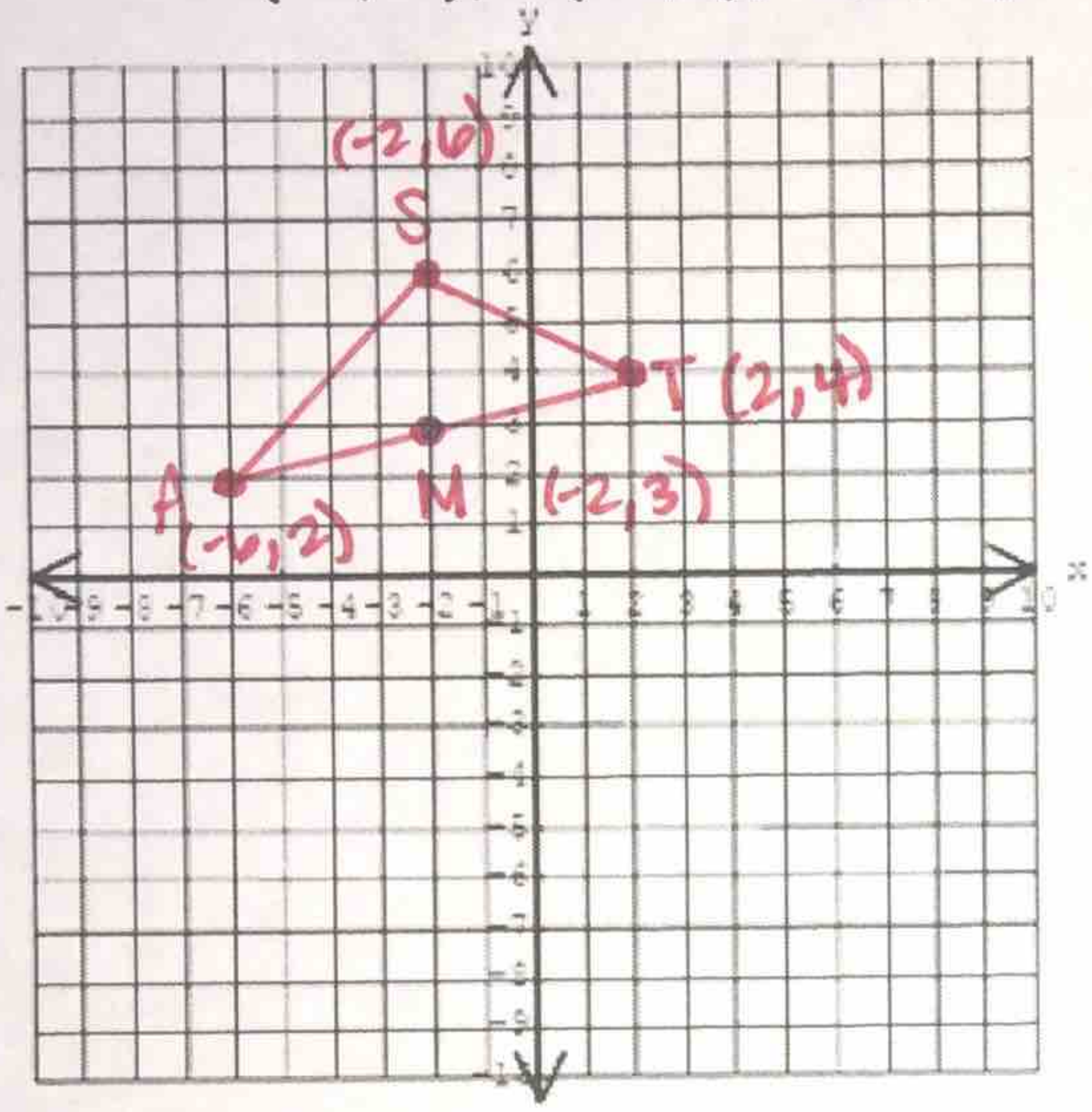
Right triangle
P is on triangle.



Obtuse triangle
P is outside triangle.

In an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude from the vertex angle to the base angle are all the same segment. In an equilateral triangle, this is true from any vertex.

Ex 4: Find the coordinates of the centroid P of Δ ABC with A(-6, 2), S(-2,6), T(2, 4).



$$M_{AT} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-6+2}{2}, \frac{2+4}{2}\right)$$

$$= (-2, 3)$$

SM = 3 units

$SP = \frac{2}{3} SM$

$SP = \frac{2}{3}(3)$

SP = 2 units (down from S)

$\boxed{P(-2,4)}$

The Measures of Center in Triangles:

- **Circumcenter** - point of concurrency of segment bisectors; inside for acute Δs, on the triangle for right Δs, outside for obtuse Δs
- **Incenter** - point of concurrency of angle bisectors; inside Δs always
- **Centroid** - point of concurrency of medians; inside Δs always
- **Orthocenter** - point of concurrency of altitudes; inside for acute Δs, on the triangle for right Δs, outside for obtuse Δs