5.3 Use Angle Bisectors of Triangles

angle bisector - a ray that divides an angle into 2 congruent adjacent angles

distance from a point to a line - the length of the perpendicular segment from the point to the line

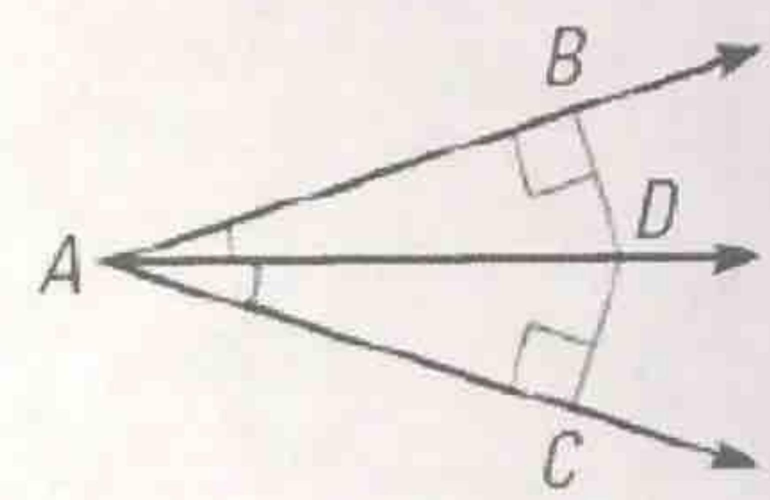
THEOREMS *For Your Notebook*

THEOREM 5.5 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.

Proof: Ex. 34, p. 315

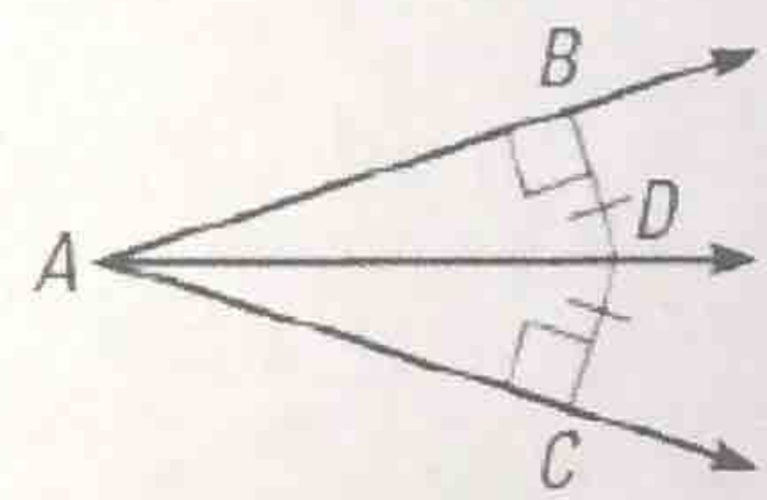


THEOREM 5.6 Converse of the Angle Bisector Theorem

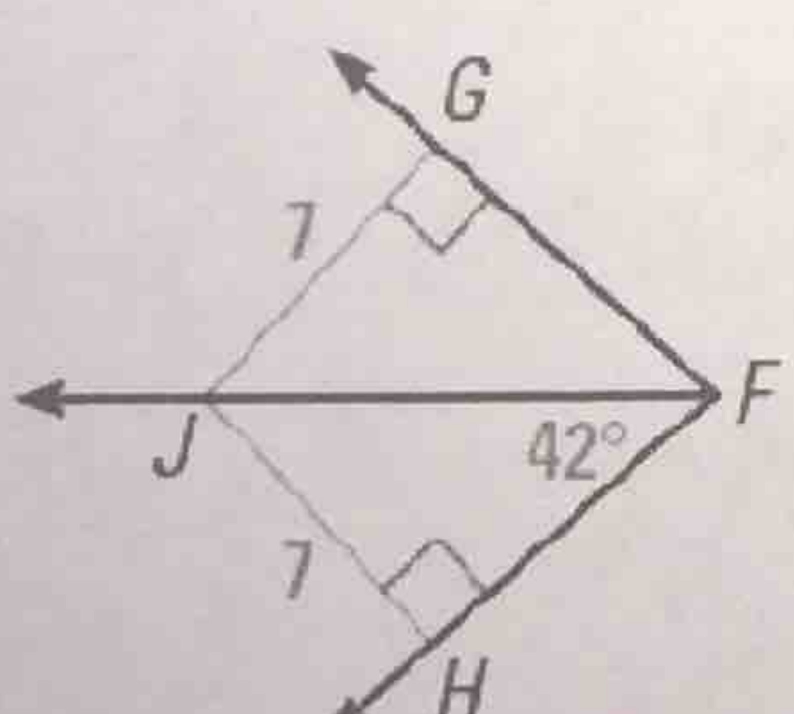
If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

Proof: Ex. 35, p. 315



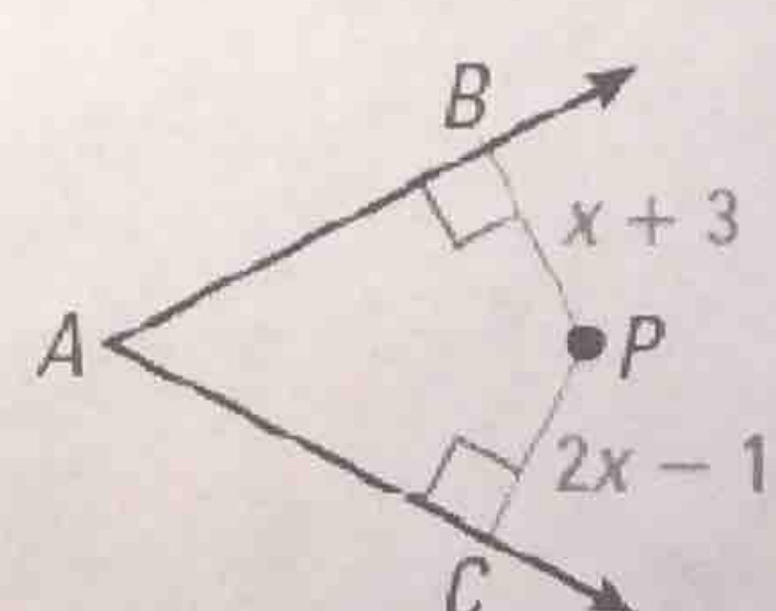
Ex 1: Find the measure of $\angle GFJ$.



$\overrightarrow{FJ}$ Bisects $\angle GFH$ by Converse of the Angle Bisector

$m\angle GFJ = m\angle HFJ = \boxed{42^\circ}$

Ex 2: For what value of **x** does **P** lie on the bisector of $\angle A$?



$BP = CP$ by Converse of the Angle Bisector Theorem

$x + 3 = 2x - 1$

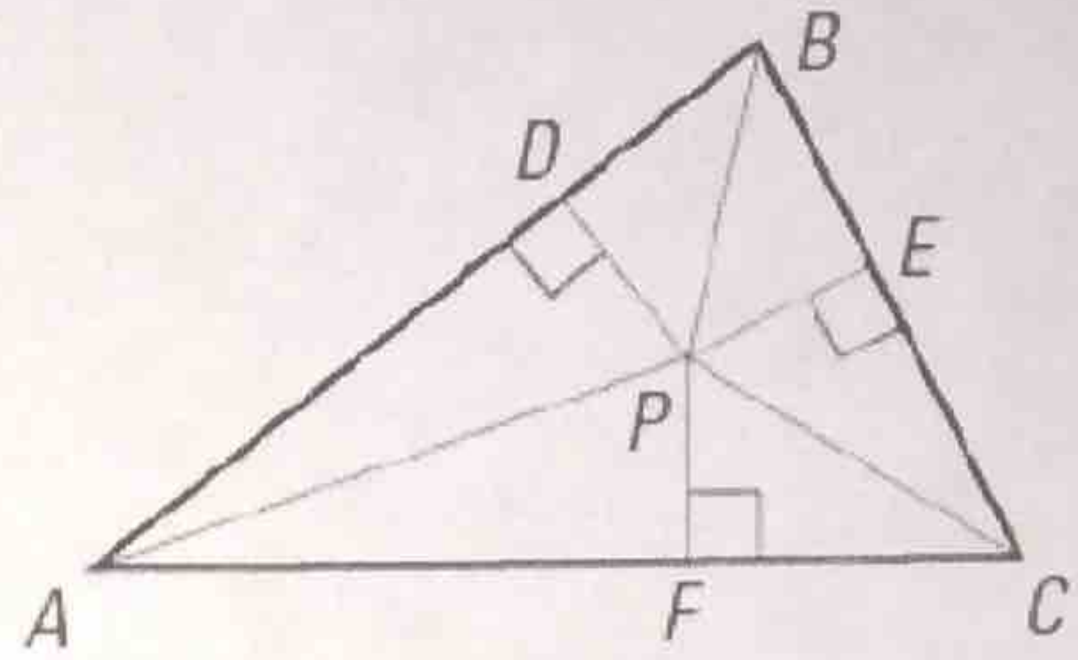
$\boxed{x = 4}$

THEOREM — *For Your Notebook*

THEOREM 5.7 Concurrency of Angle Bisectors of a Triangle

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

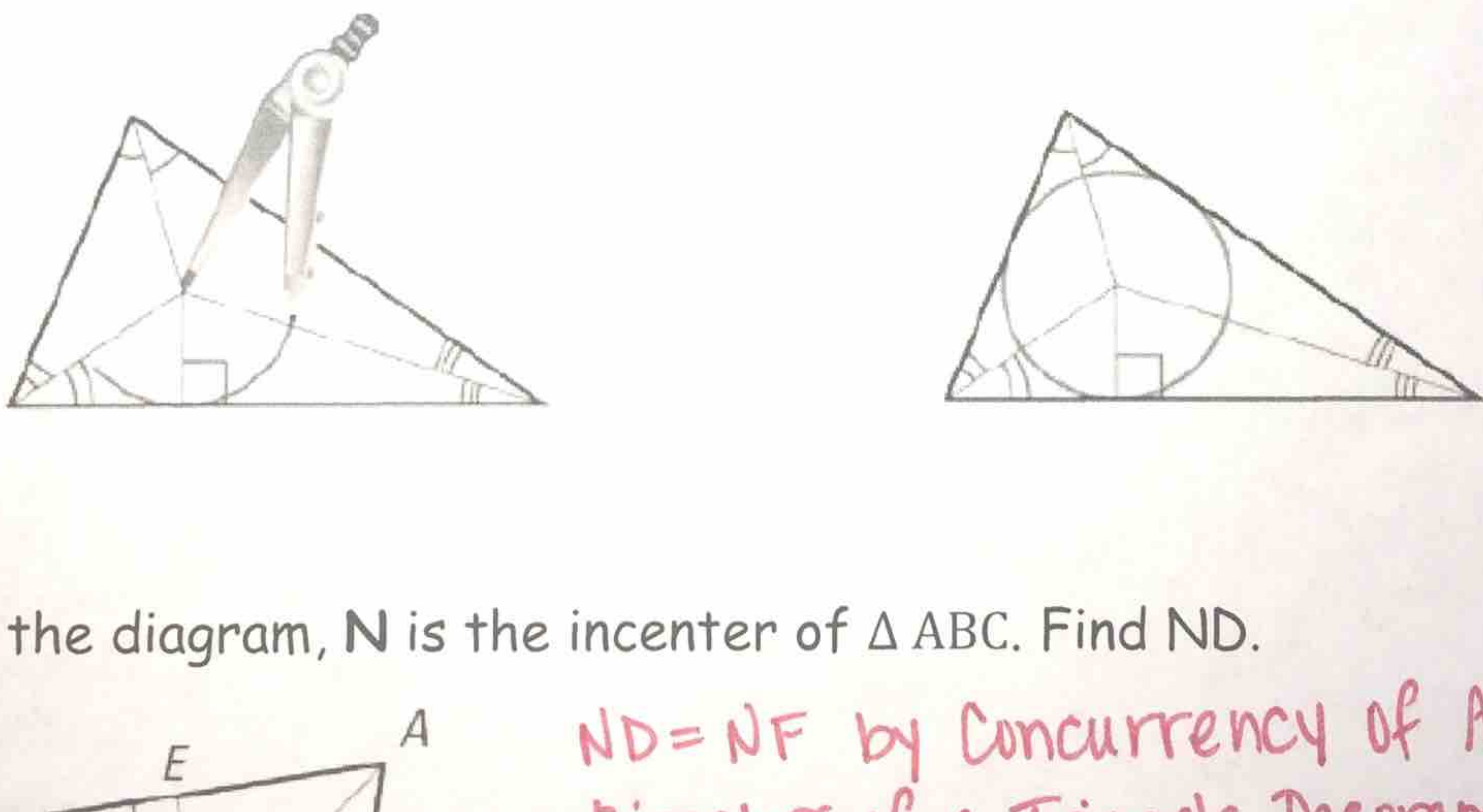
If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.



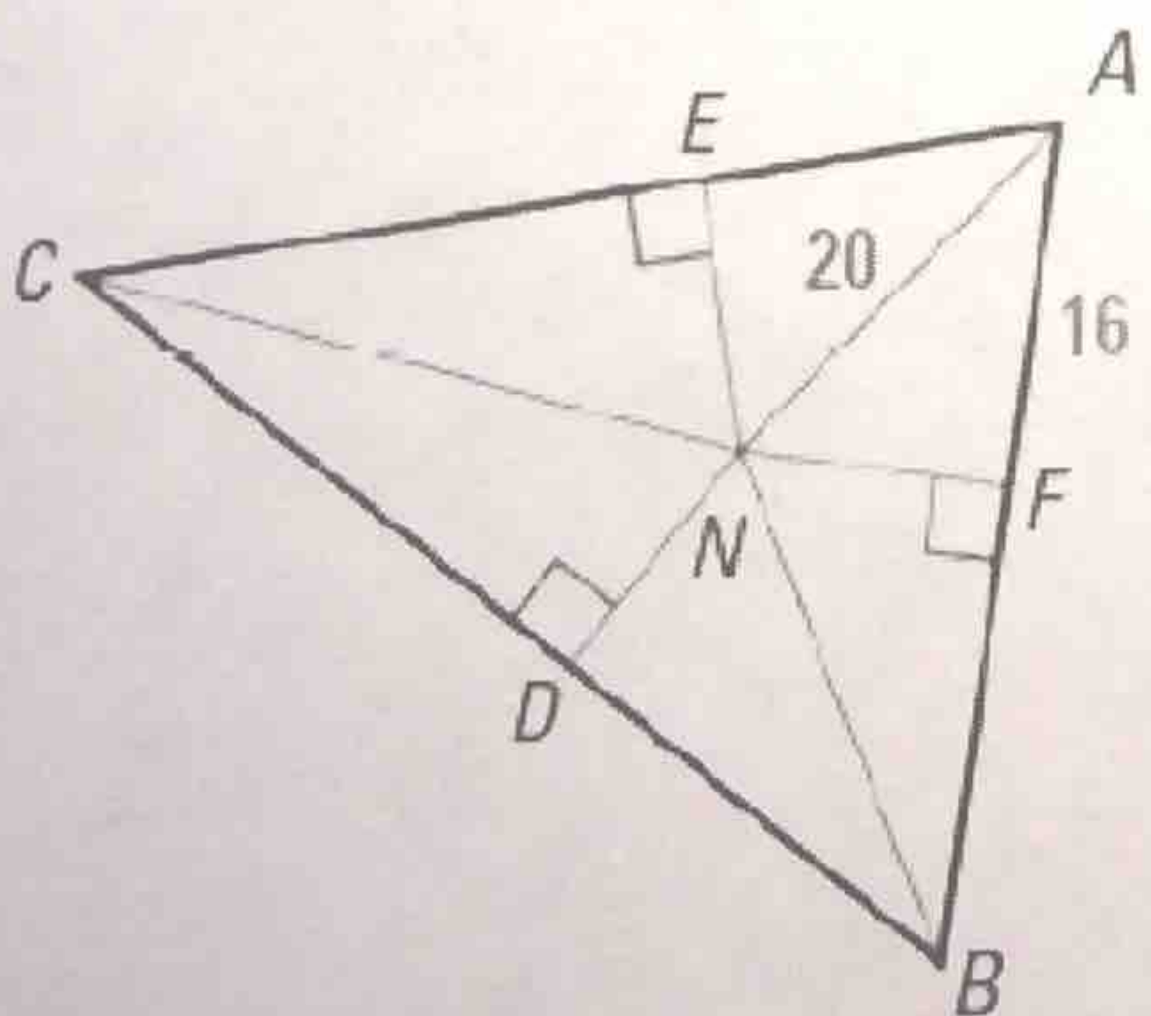
Proof: Ex. 36, p. 316

INCENTER – the point of concurrency of the 3 angle bisectors of a triangle, it always lies inside the triangle

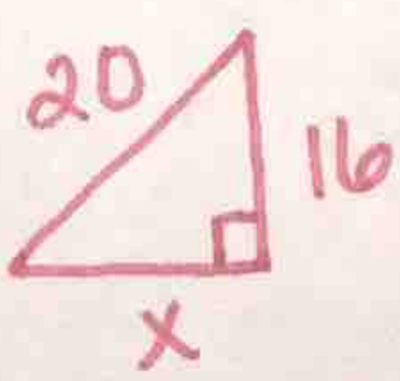
Because the incenter **P** is equidistant from the 3 sides of the triangle, a circle drawn using **P** as the center and the distance to one side as the radius will just touch the other 2 sides. The circle is said to be **inscribed** within the triangle.



Ex 3: In the diagram, **N** is the incenter of $\triangle ABC$. Find ND.



ND = NF by Concurrency of Angle Bisectors of a Triangle Theorem



$a^2 + b^2 = c^2$

$x^2 + 16^2 = 20^2$

$x^2 + 256 = 400$

$x^2 = 144$

$x = 12$

$\boxed{ND = NF = 12 \text{ units}}$