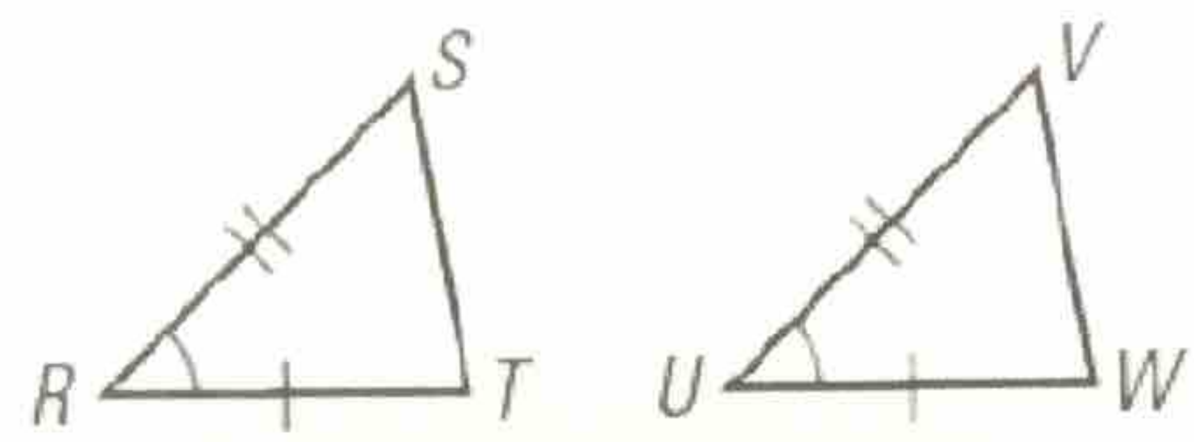
4.4 Prove Triangles Congruent by SAS and HL

POSTULATE *For Your Notebook*

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If Side $\overline{RS} \cong \overline{UV}$,
Angle $\angle R \cong \angle U$, and
Side $\overline{RT} \cong \overline{UW}$,
then $\triangle RST \cong \triangle UVW$.

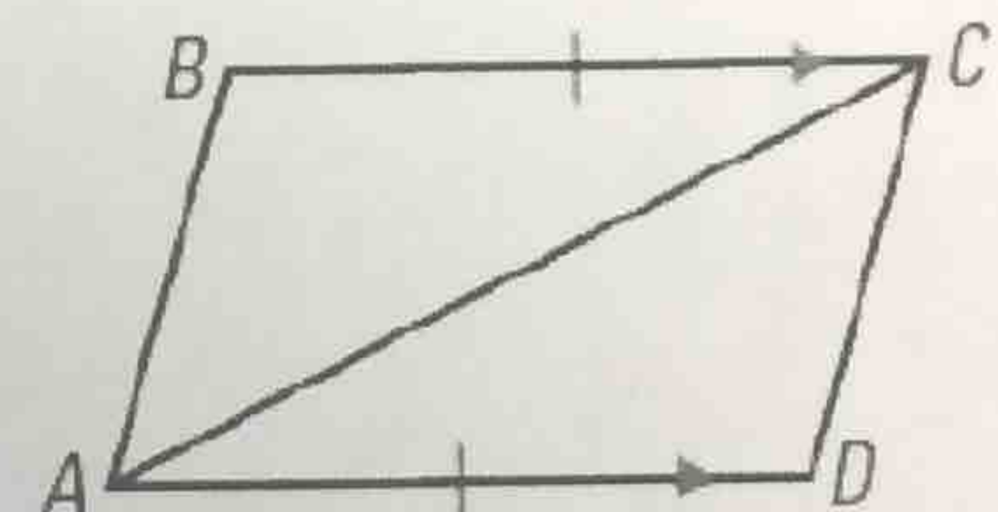


Ex 1:

Write a proof.

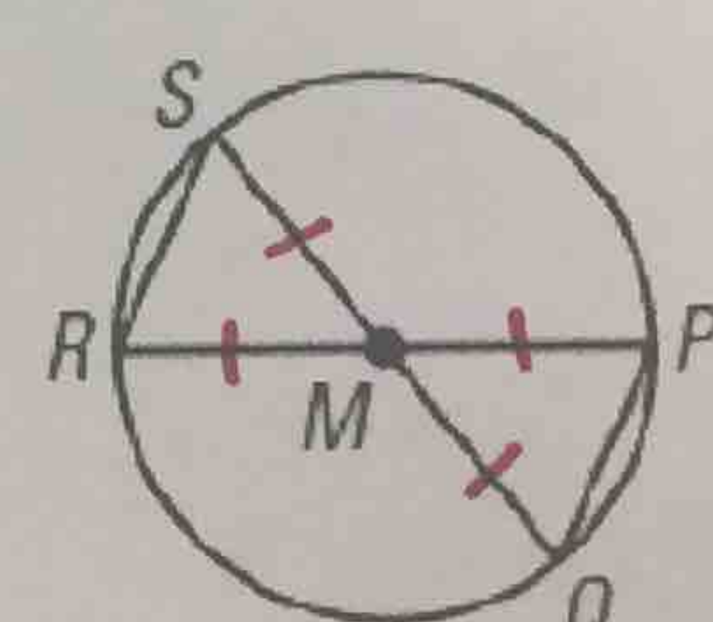
GIVEN ▶ $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\triangle ABC \cong \triangle CDA$



STATEMENTS	REASONS
S 1. $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$	1. Given
A 2. $\angle BCA \cong \angle DAC$	2. Alternate Interior Angles Theorem
S 3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Property of Congruence
4. $\triangle ABC \cong \triangle CDA$	4. SAS

Ex 2: In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?



$\overline{MP} \cong \overline{MQ} \cong \overline{MR} \cong \overline{MS}$

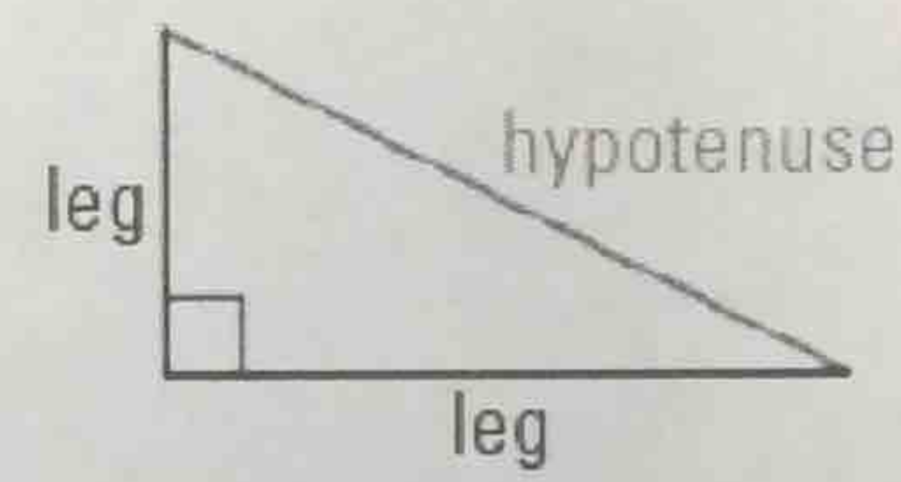
$\angle PMQ \cong \angle RMS$

$\triangle MRS \cong \triangle MPQ$: SAS

In general, if you know the lengths of two sides and the measure of an angle that is *not included* between them, you can create two different triangles:

Therefore, SSA is **not** a valid method for proving that triangles are congruent, although there is a special case for **right triangles**.

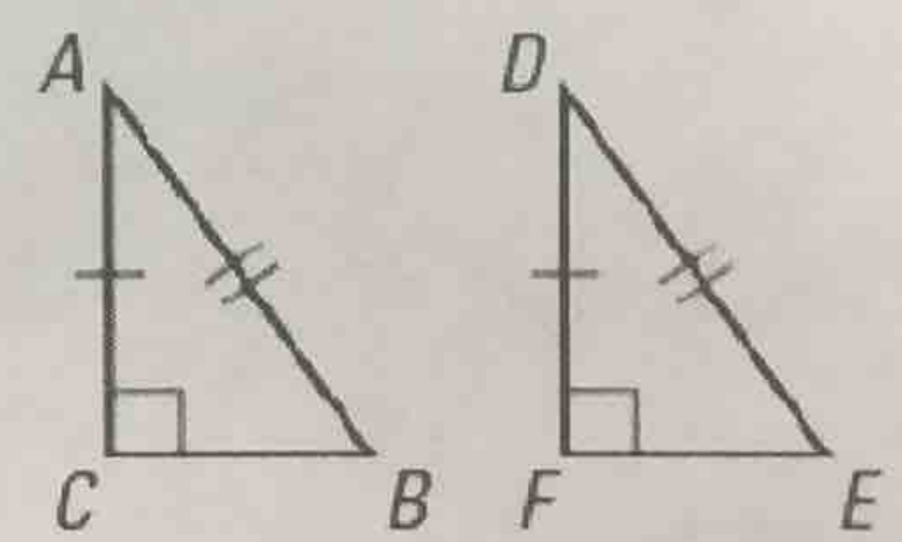
<u>right triangle</u> – a triangle with a right angle, the sides adjacent to the right angle are called **legs**, the side opposite the right angle is called the **hypotenuse** of the right triangle



THEOREM *For Your Notebook*

THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

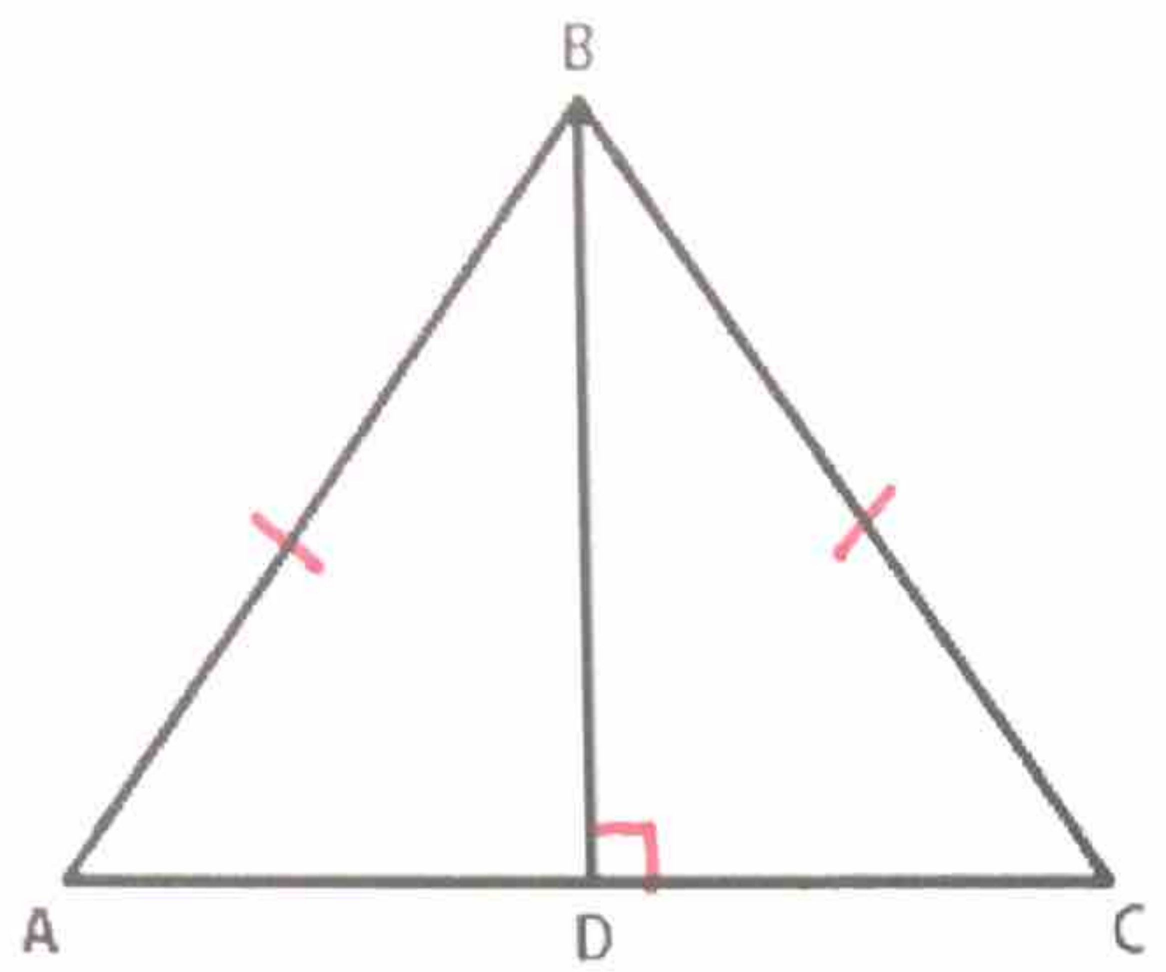


$\triangle ABC \cong \triangle DEF$

Proofs: Ex. 37, p. 439; p. 932

Ex 3: GIVEN: $\overline{BD}$ is perpendicular to $\overline{AC}$, $\overline{AB} \cong \overline{CB}$

PROVE: $\Delta ABD \cong \Delta CBD$



STATEMENTS	REASONS
1. $\overline{BD} \perp \overline{AC}$, $\overline{AB} \cong \overline{CB}$	1. Given
2. $\angle ADB$ & $\angle CDB$ are right angles	2. Theorem 3.9 ($\perp$ lines form 4 right $\angle$s)
3. ΔABD and ΔCBD are right Δs	3. Definition of right triangle
4. $\overline{BD} \cong \overline{BD}$	4. Reflexive Property of Congruence
5. $\Delta ABD \cong \Delta CBD$	5. HL

Things to remember:

- Regular polygon – all sides are congruent, all angles are congruent
- Circle – all radii are congruent
- Square – four congruent sides, four congruent angles