

4.1 Apply Triangle Sum Properties

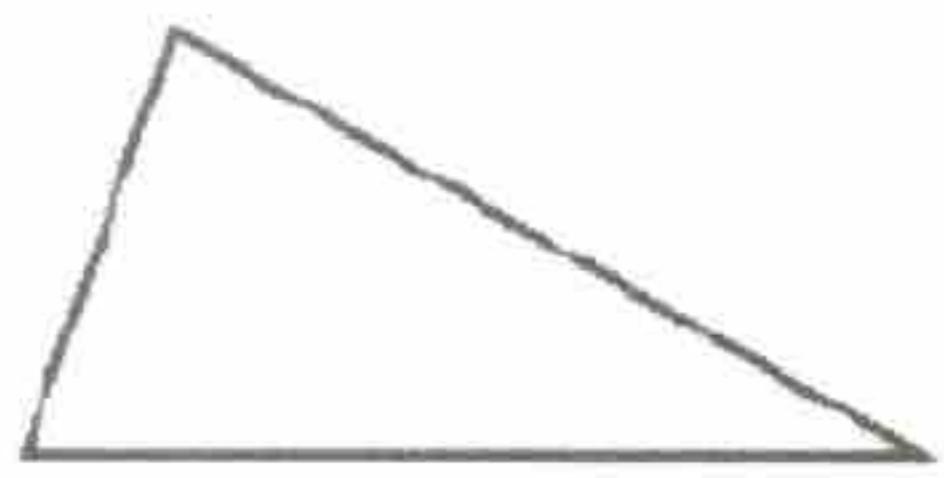
triangle - a polygon with three sides

KEY CONCEPT

For Your Notebook

Classifying Triangles by Sides

Scalene Triangle



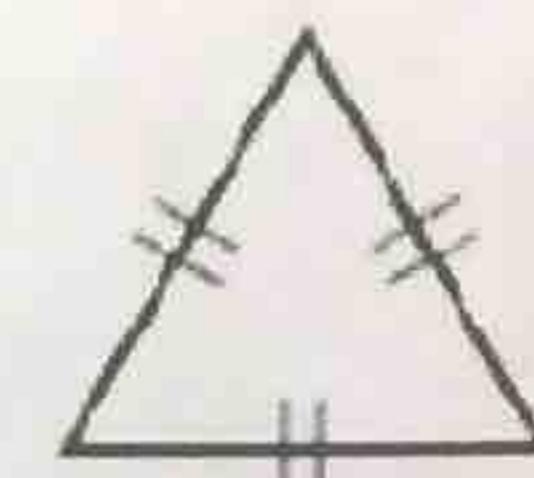
No congruent sides

Isosceles Triangle



At least 2 congruent sides

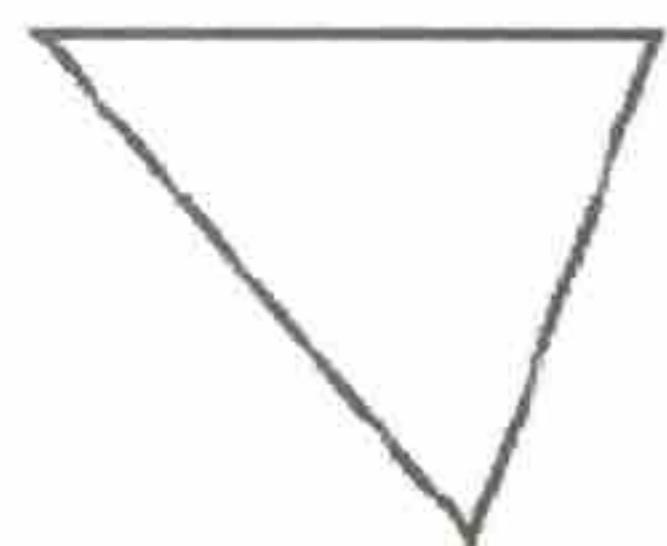
Equilateral Triangle



3 congruent sides

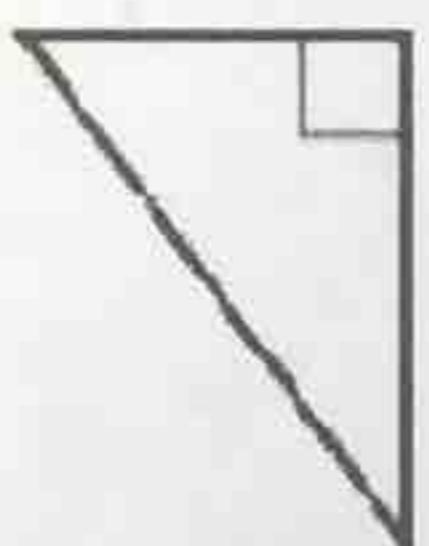
Classifying Triangles by Angles

Acute Triangle



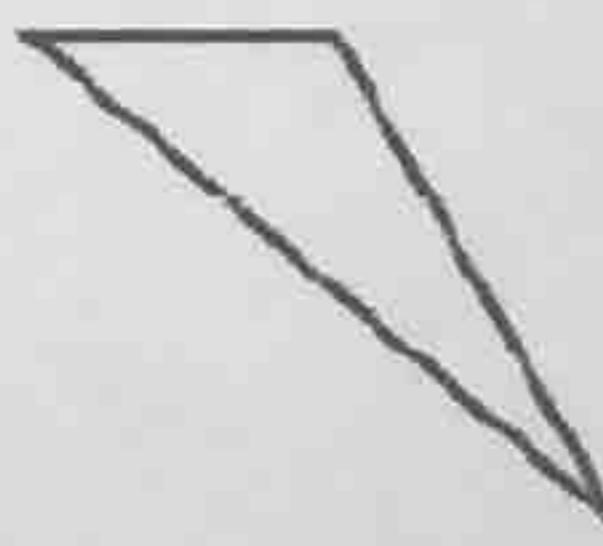
3 acute angles

Right Triangle



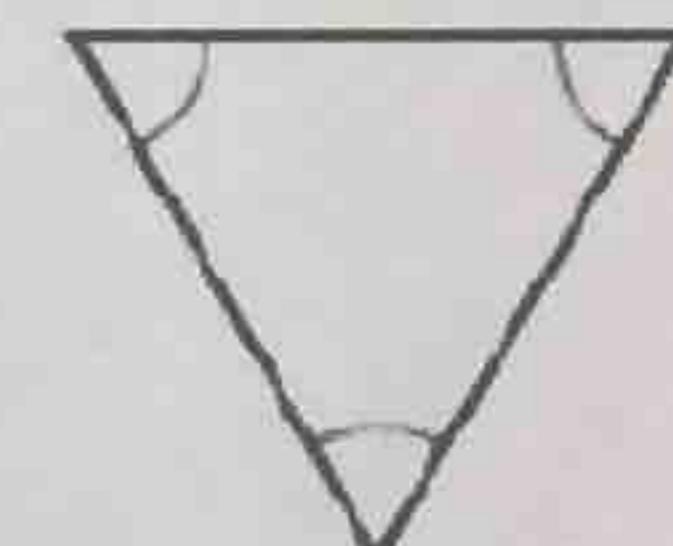
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle



3 congruent angles

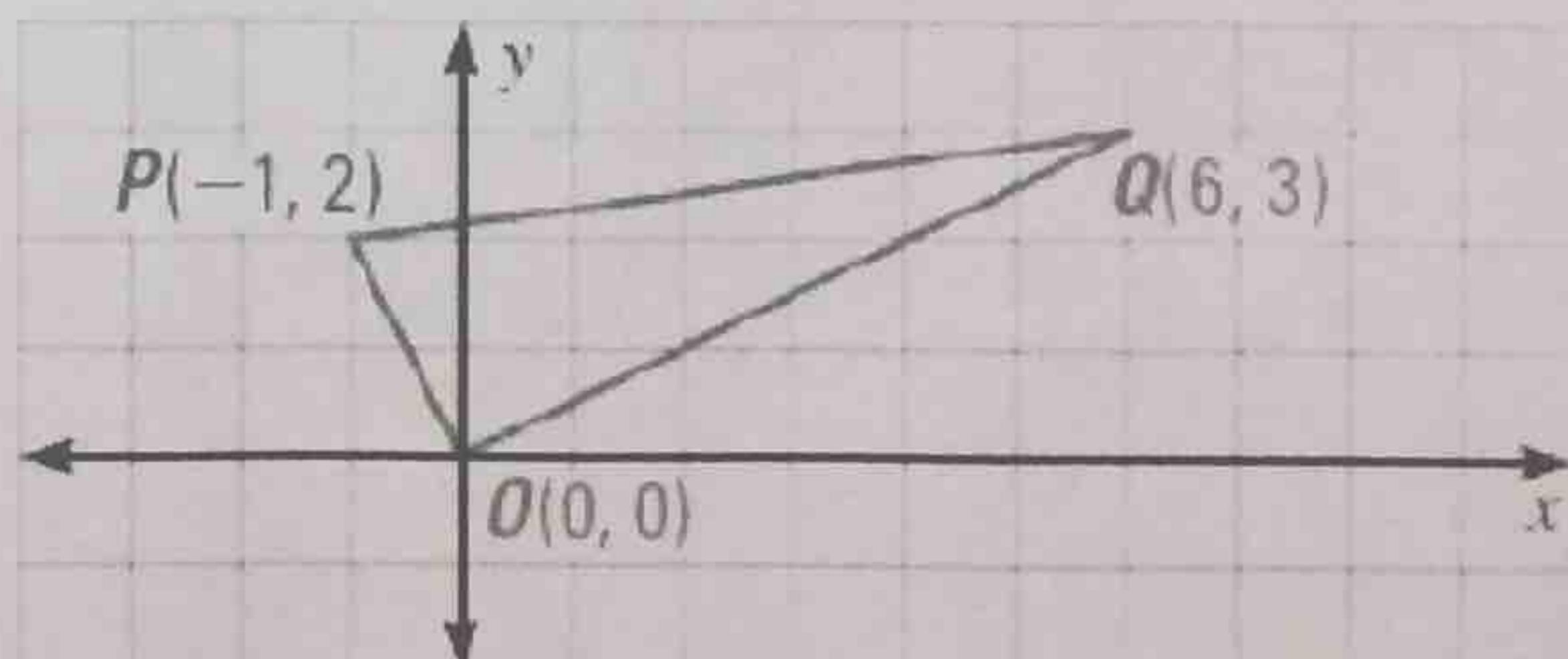
Ex 1: Classify $\triangle PQR$ by its sides. Then determine whether the triangle is a right triangle.

$$\begin{aligned} d_{OP} &= \sqrt{((-1)-0)^2 + (2-0)^2} \\ &= \sqrt{1^2 + 2^2} \\ &= \sqrt{5} \approx 2.2 \end{aligned}$$

$$\begin{aligned} d_{OQ} &= \sqrt{(6-0)^2 + (3-0)^2} \\ &= \sqrt{6^2 + 3^2} \\ &= \sqrt{45} \approx 6.7 \end{aligned}$$

$$\begin{aligned} d_{PQ} &= \sqrt{(6-(-1))^2 + (3-2)^2} \\ &= \sqrt{7^2 + 1^2} \\ &= \sqrt{50} \approx 7.1 \end{aligned}$$

scalene



$$m_{OP} = \frac{2-0}{-1-0} = \frac{2}{-1} = -2$$

$$m_{OQ} = \frac{3-0}{6-0} = \frac{3}{6} = \frac{1}{2}$$

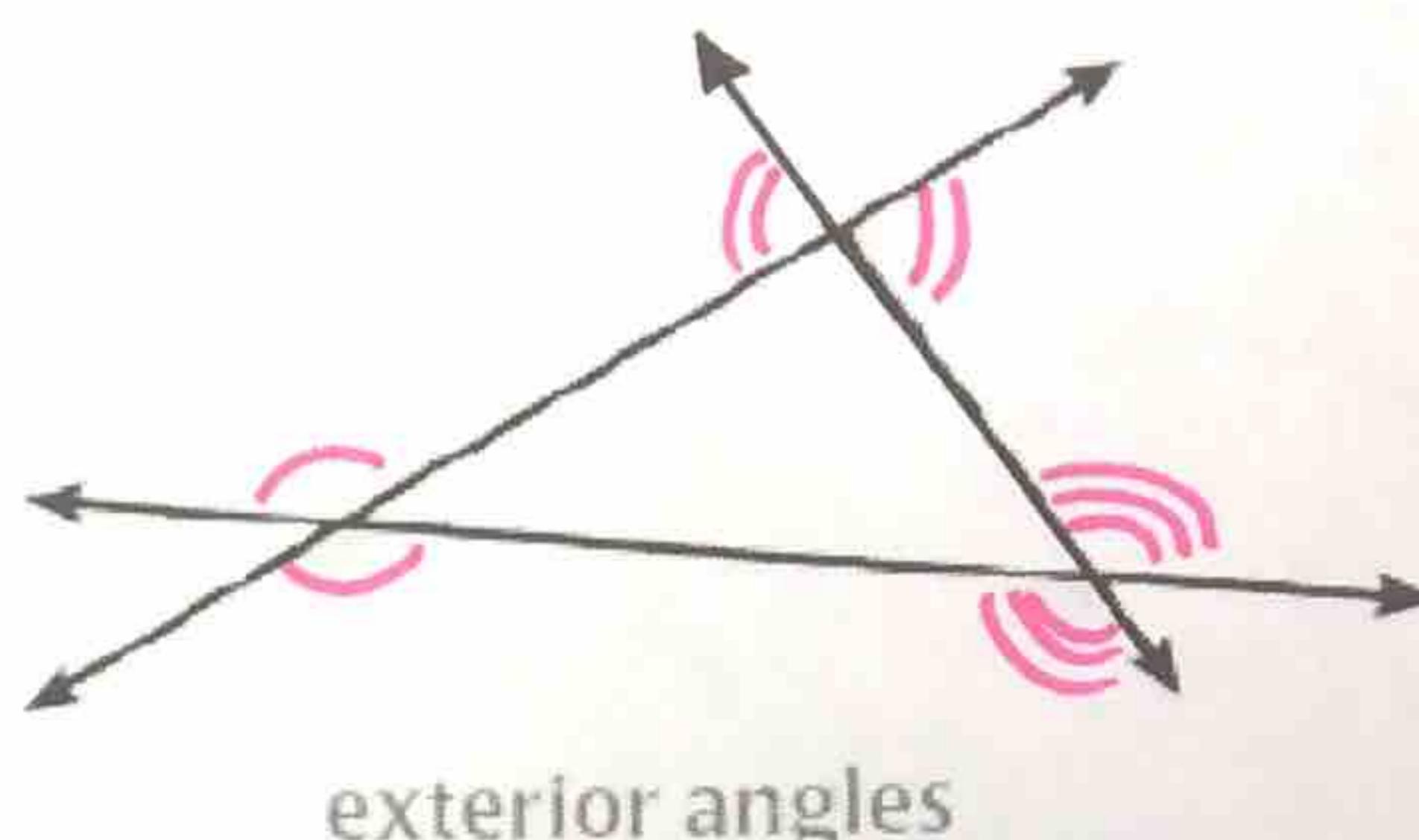
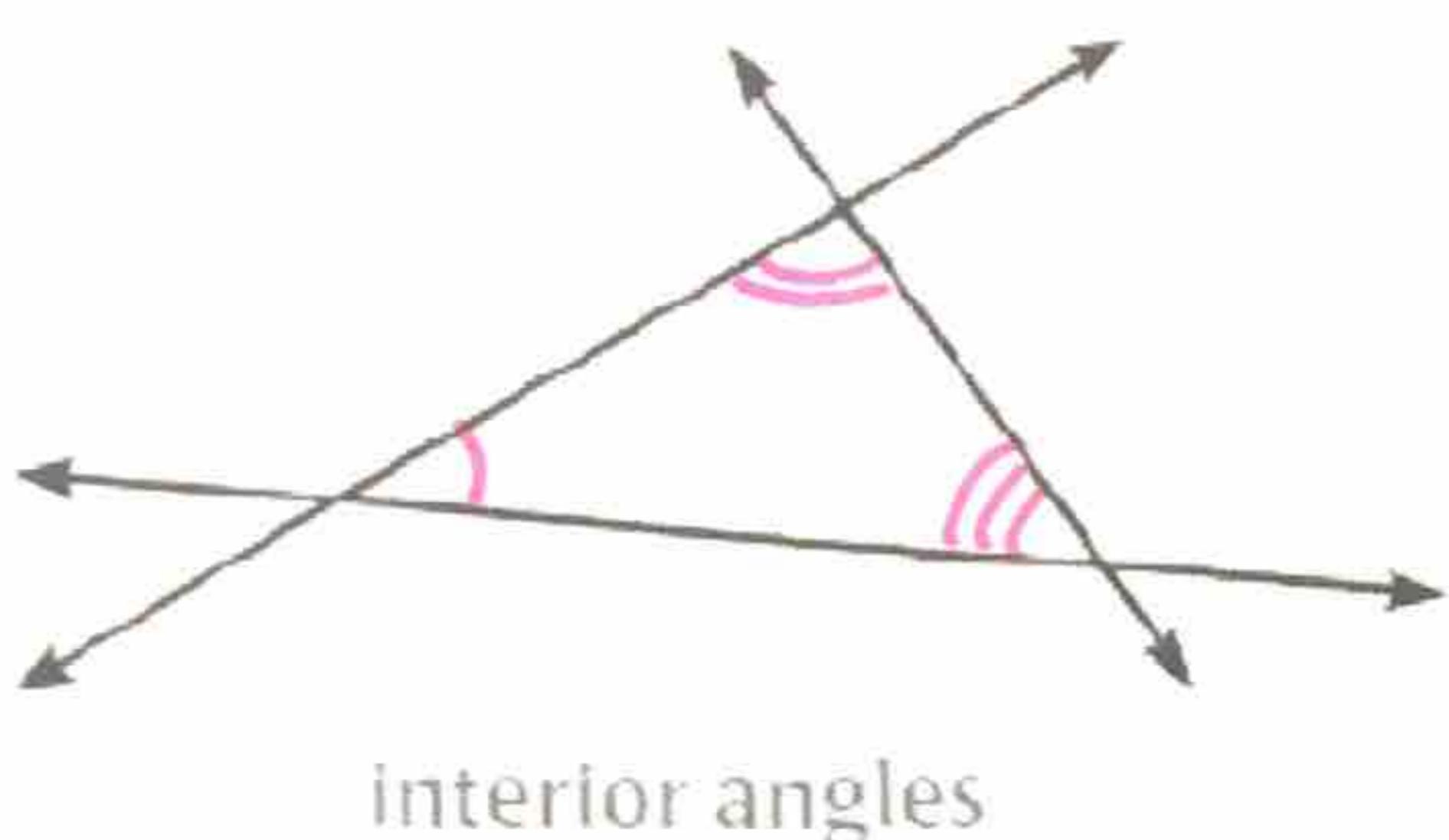
$$m_{PQ} = \frac{3-2}{6+1} = \frac{1}{7}$$

right scalene \triangle

or
Pythagorean
Theorem!

interior angles - the original angles inside of a polygon

exterior angles - when the sides of a polygon are extended, the angles that form linear pairs with the interior angles



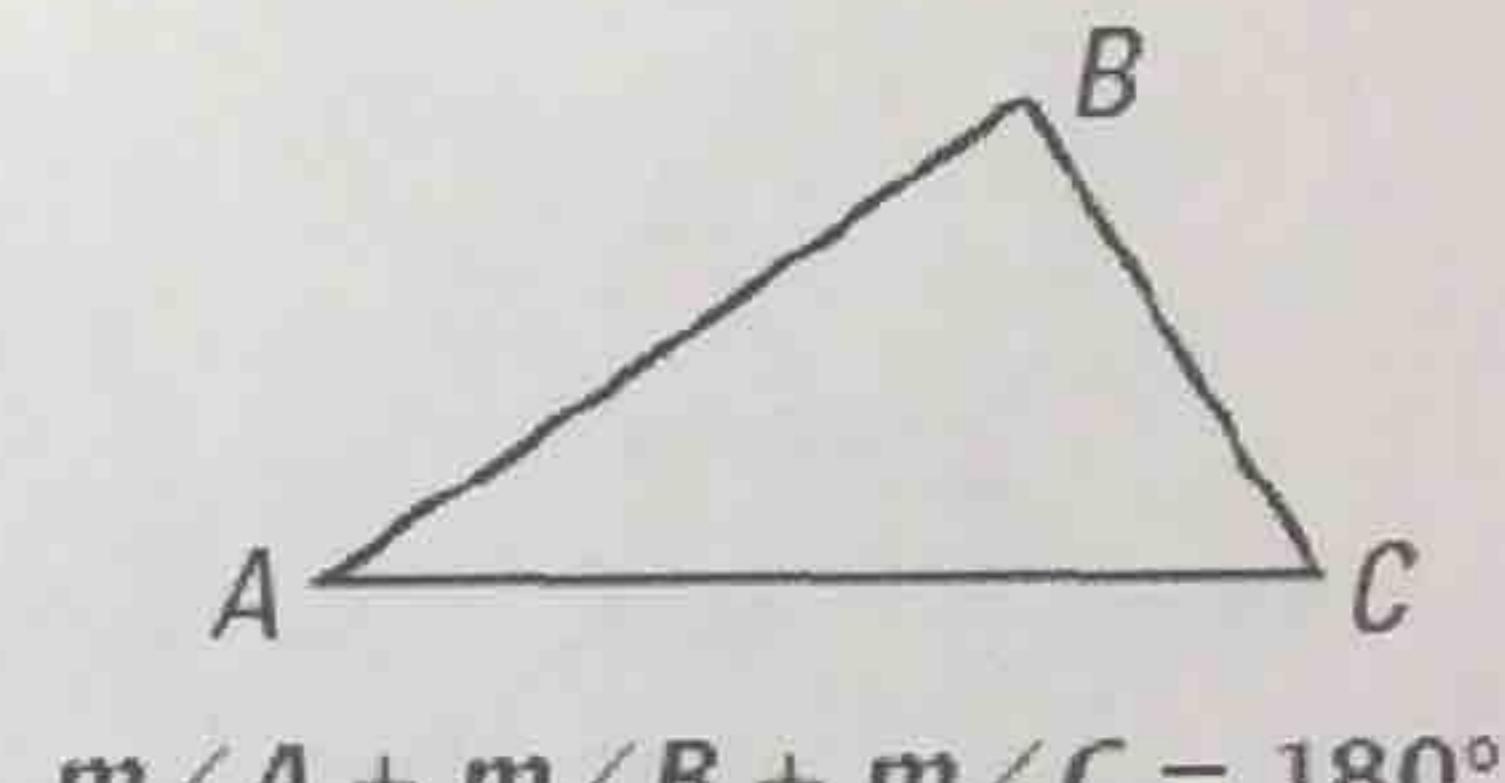
THEOREM

For Your Notebook

THEOREM 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180° .

Proof: p. 219; Ex. 53, p. 224



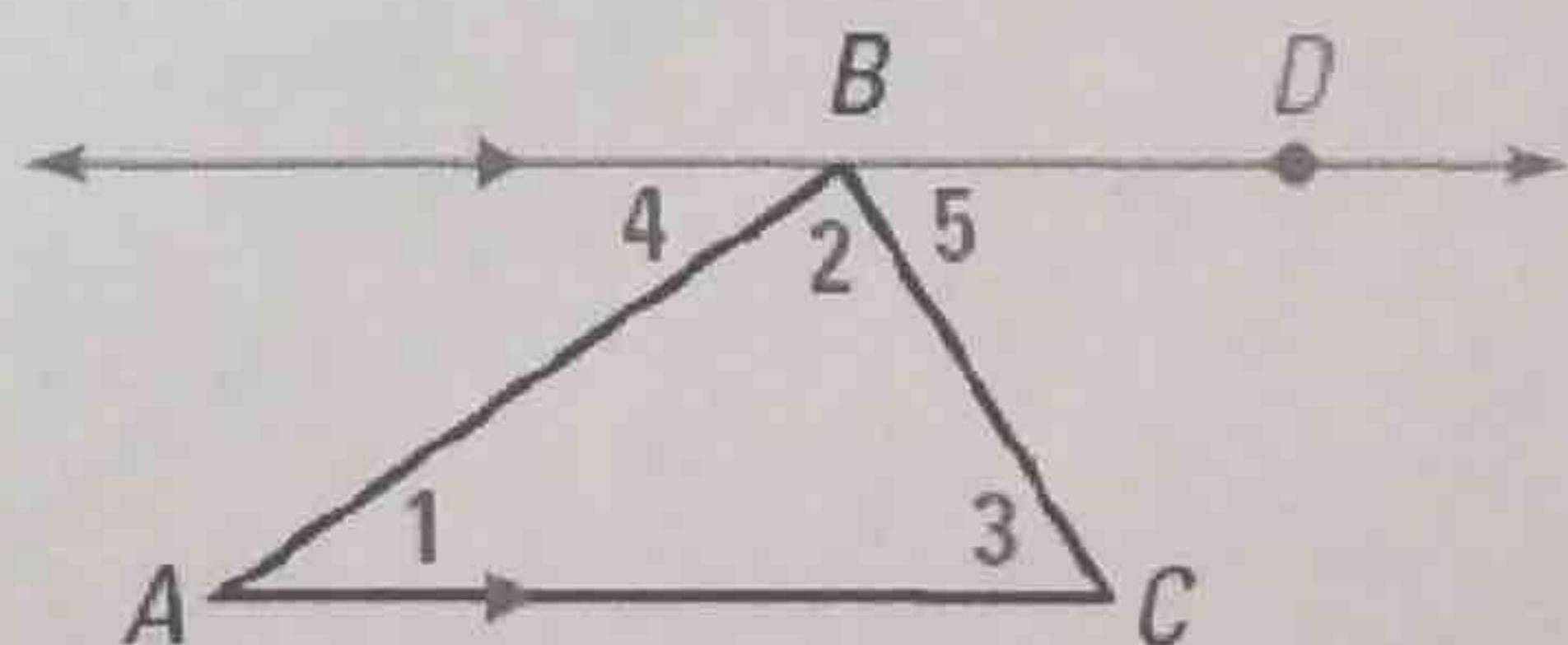
$$m\angle A + m\angle B + m\angle C = 180^\circ$$

auxiliary lines - to prove certain theorems, a line, line segment, or ray may need to be added to a given diagram

Ex 2: Prove the Triangle Sum Theorem

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$



STATEMENTS

1. $\overleftrightarrow{BD} \parallel \overline{AC}$
2. $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$
3. $\angle 1 \cong \angle 4, \angle 3 \cong \angle 5$
4. $m\angle 1 = m\angle 4, m\angle 3 = m\angle 5$
5. $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$

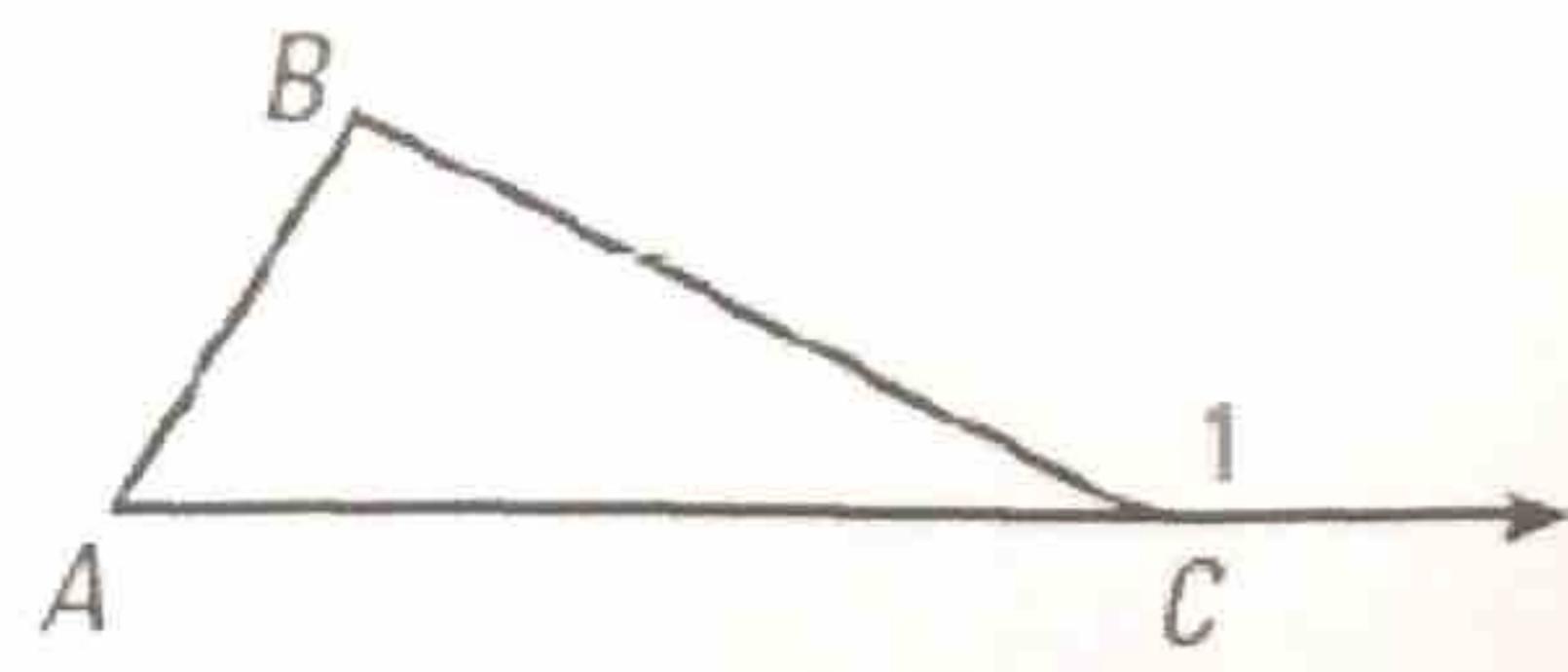
REASONS

1. Definition of parallel
2. Angle Addition Postulate & Defn. of straight
3. Alternate Interior Angles Theorem
4. Definition of Congruent Angles
5. Substitution Property of Equality

THEOREM**For Your Notebook****THEOREM 4.2 Exterior Angle Theorem**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Proof: Ex. 50, p. 223



$$m\angle 1 = m\angle A + m\angle B$$

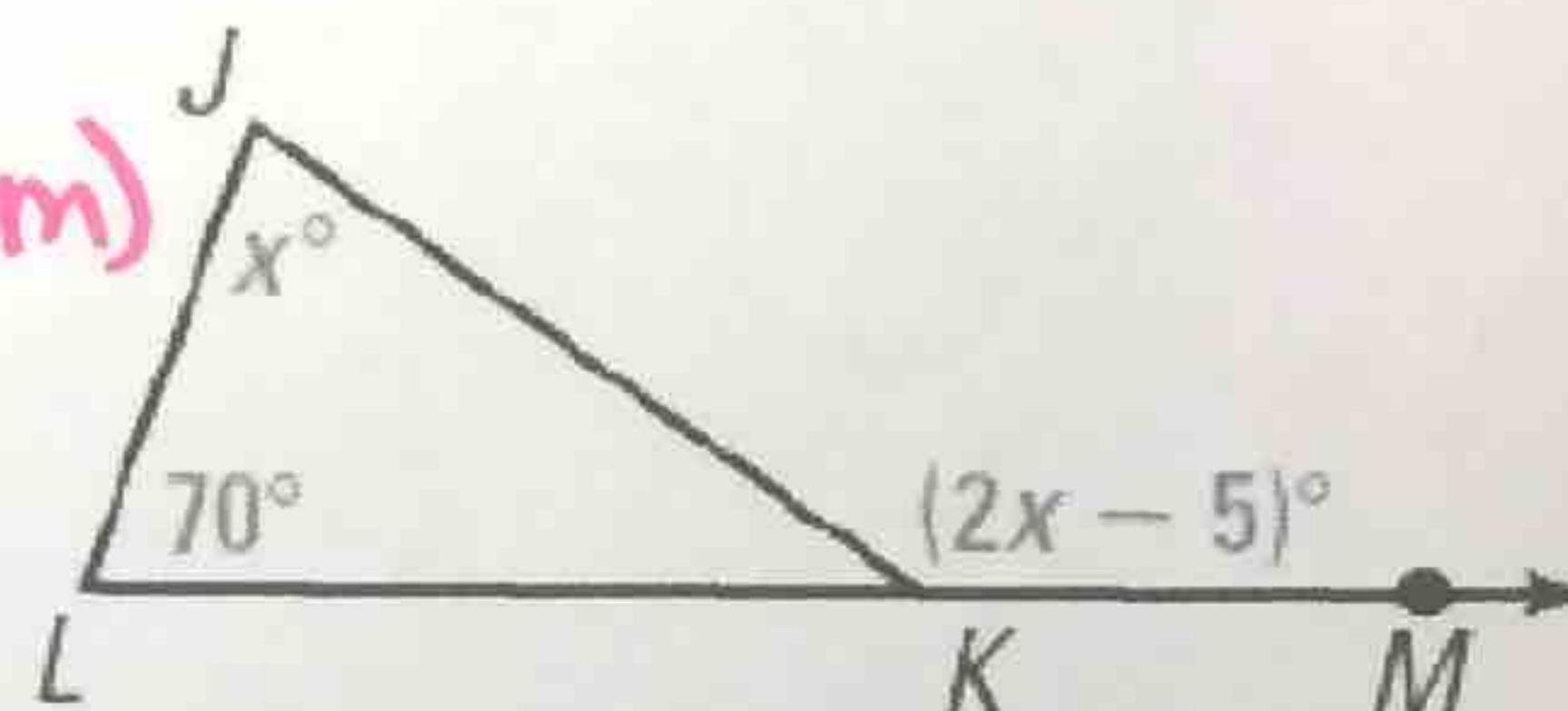
Ex 3: Find $m\angle JKM$.

$$2x - 5 = 70 + x \quad (\text{Ext. Angle Thm})$$

$$x = 75$$

$$\begin{aligned} m\angle JKM &= 2x - 5 \\ &= 2(75) - 5 \\ &= 150 - 5 \end{aligned}$$

$$\boxed{m\angle JKM = 145^\circ}$$

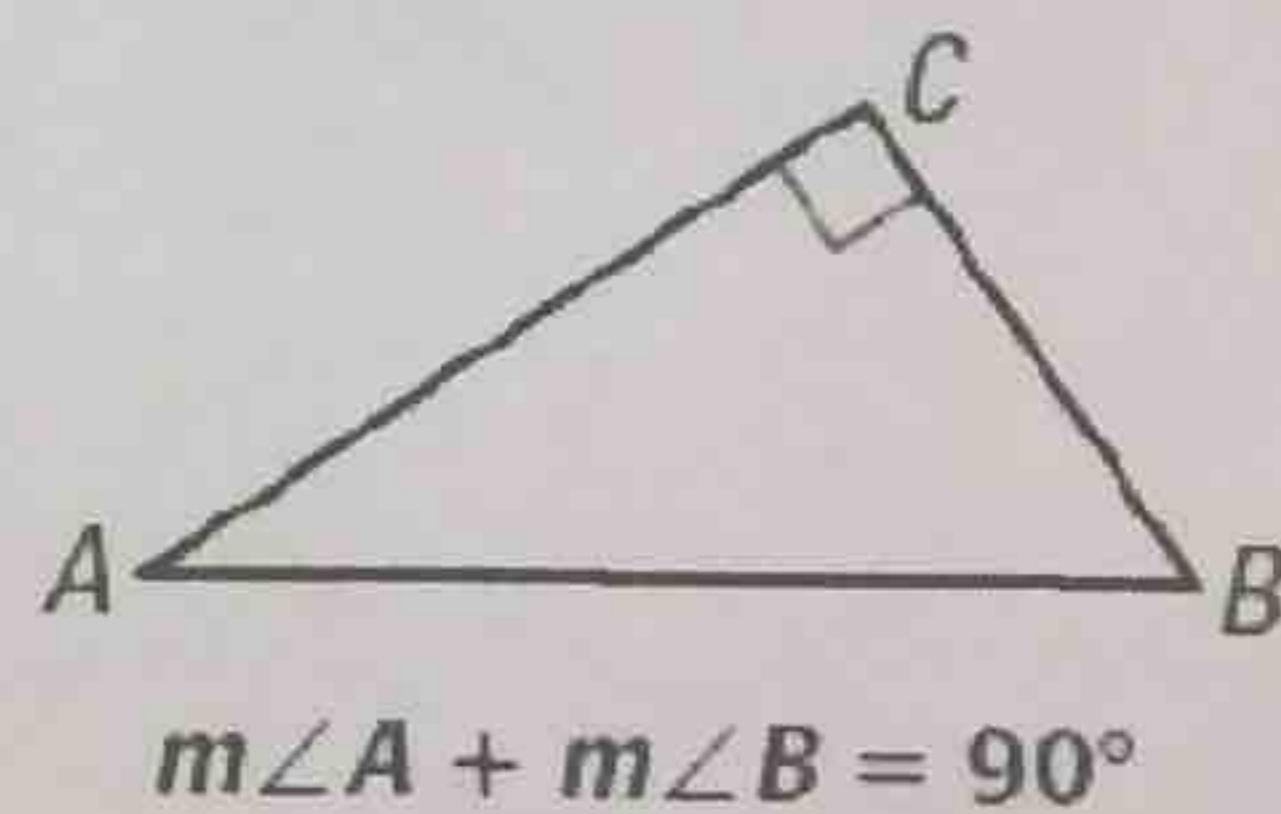


corollary to a theorem - a statement that can be proved easily using a certain theorem

COROLLARY**For Your Notebook****Corollary to the Triangle Sum Theorem**

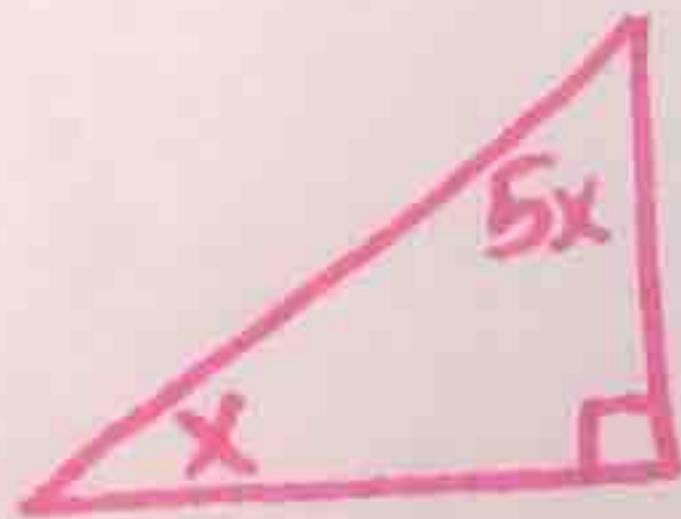
The acute angles of a right triangle are complementary.

Proof: Ex. 48, p. 223



$$m\angle A + m\angle B = 90^\circ$$

Ex 4: The support for a skateboard ramp forms a right triangle. The measure of one acute angle in the triangle is five times the measure of the other. Find the measure of each acute angle.



$$\begin{aligned} 5x + x &= 90 \\ 6x &= 90 \\ x &= 15 \end{aligned} \quad (\text{Corollary to } \Delta \text{ Sum Theorem})$$

$$\boxed{15^\circ, 75^\circ}$$