2.5 Reason Using Properties from Algebra

Segment lengths and angle measures are real numbers, so we can use the properties of Algebra to write logical arguments about geometric figures.

KEY CONCEPT *For Your Notebook*

Algebraic Properties of Equality

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression.

Ex 1: Solve 3x + 8 = -4x - 34. Write a reason for each step.

$3x+8=-4x-34$ Given

$3x+8+4x=-4x-34+4x$ Addition Property of Equality

$7x+8=-34$ Simplify

$7x+8-8=-34-8$ Subtraction Property of Equality

$7x=-42$ Simplify

$x=-6$ Division Property of Equality

$\boxed{\text{The value of } x \text{ is } -6.}$

KEY CONCEPT *For Your Notebook*

Distributive Property

$a(b + c) = ab + ac$, where a, b, and c are real numbers.

Ex 2: Solve 60 = -3(8x - 4). Write a reason for each step.

$60 = -3(8x-4)$	Given
$60 = -24x + 12$	Distributive Property
$48 = -24x$	Subtraction Property of Equality
$-2 = x$	Division Property of Equality

$\boxed{\text{The value of } x \text{ is } -2.}$

Ex 3: The cost C of using a certain cell phone can be modeled by the plan rate formula C = 0.30(m - 300) + 39.99, where **m** represents the number of minutes over 300. Solve the formula for m. Write a reason for each step.

$C = 0.30(m-300) + 39.99$	Given
$C = 0.30m - 90 + 39.99$	Distributive Property
$C = 0.3m - 50.01$	Simplify
$C + 50.01 = 0.3m$	Addition Property of Equality
$\boxed{\frac{C+50.01}{0.3} = m}$	Division Property of Equality

These properties are true for all real numbers (therefore they are also true for segment lengths and angle measures):

KEY CONCEPT *For Your Notebook*

Reflexive Property of Equality

Real Numbers	For any real number a, $a = a$.
Segment Length	For any segment $\overline{AB}$, $AB = AB$.
Angle Measure	For any angle $\angle A$, $m\angle A = m\angle A$.

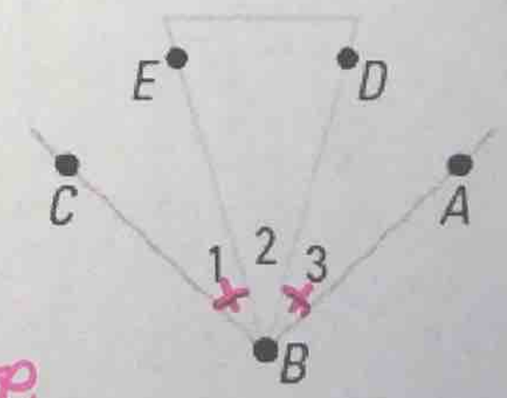
Symmetric Property of Equality

Real Numbers	For any real numbers a and b, if $a = b$, then $b = a$.
Segment Length	For any segments $\overline{AB}$ and $\overline{CD}$, if $AB = CD$, then $CD = AB$.
Angle Measure	For any angles $\angle A$ and $\angle B$, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

Real Numbers	For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Segment Length	For any segments $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, if $AB = CD$ and $CD = EF$, then $AB = EF$.
Angle Measure	For any angles $\angle A$, $\angle B$, and $\angle C$, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Ex 4: You are designing a logo to sell daffodils. Use the information given. Determine whether $m\angle EBA = m\angle DBC$. Explain your reasoning.



$m\angle 1 = m\angle 3$ Given

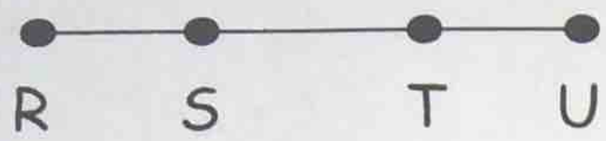
$m\angle EBA = m\angle 3 + m\angle 2$ Angle Addition Postulate

$m\angle EBA = m\angle 1 + m\angle 2$ Substitution Prop. of Eq.

$m\angle 1 + m\angle 2 = m\angle DBC$ Angle Addition Postulate

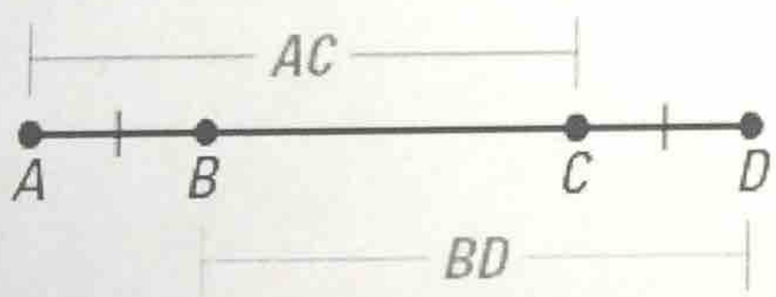
$m\angle EBA = m\angle DBC$ Transitive Property of Equality

Ex 5: The city is planning to add two stations between the beginning and end of a commuter train line. Use the information given: RT = SU. Determine whether RS = TU. Explain your reasoning.



RT = SU Given
ST = ST Reflexive Property
RT − ST = SU − ST Subtraction Prop. of Eq.
RT − ST = RS Segment Addition Postulate
SU − ST = TU Segment Add. Post.
RS = TU Substitution Prop. of Equality

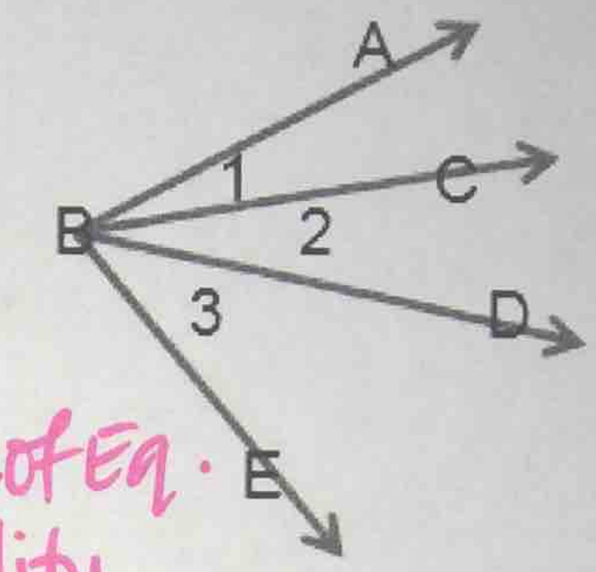
Ex 6: In the diagram, AB = CD. Show that AC = BD. Explain your reasoning.



AB = CD Given
AC = AB + BC Segment Addition Postulate
BD = BC + CD Segment Addition Postulate
AB + BC = CD + BC Add. Prop. of Equality
AC = BD Substitution Prop. of Equal.

AB = CD Given
BC = BC Reflexive
AB + BC = CD + BC Add. Prop. of Equal.
AC = AB + BC Segment Add. Post.
BD = BC + CD segment Add. Post.
AC = BD Substitution

Ex 7: In the diagram $m\angle ABD = m\angle CBE$. Show that $m\angle 1 = m\angle 3$.



$m\angle ABD = m\angle CBE$ Given
$m\angle ABD - m\angle 2 = m\angle 1$ Angle Add. Post.
$m\angle CBE - m\angle 2 = m\angle 3$ Angle Add. Post.
$m\angle ABD - m\angle 2 = m\angle CBE - m\angle 2$ Subs. Prop. of Eq.
$m\angle 1 = m\angle 3$ Substitution Prop. of Equality

$m\angle ABD = m\angle CBE$ Given
$m\angle 2 = m\angle 2$ Reflexive
$m\angle ABD = m\angle 1 + m\angle 2$ Angle Addition Postulate
$m\angle CBE = m\angle 2 + m\angle 3$ Angle Addition Postulate
$m\angle ABD - m\angle 2 = m\angle CBE - m\angle 2$ Subtraction Prop. of Equality
$m\angle 1 = m\angle 3$ Substitution

$m\angle ABD = m\angle CBE$ Given
$m\angle 2 = m\angle 2$ Reflexive
$m\angle ABD = m\angle 1 + m\angle 2$ Angle Addition Postulate
$m\angle CBE = m\angle 2 + m\angle 3$ Angle Addition Postulate
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ Substitution
$m\angle 1 = m\angle 3$ Subtraction Prop. of Equality