2.2 Analyze Conditional Statements

conditional statement – logical statement that has 2 parts, a *hypothesis* and a *conclusion*

$$\underbrace{\text{If it is raining}}_{\text{Hypothesis}}\text{, then }\underbrace{\text{there are clouds in the sky}}_{\text{Conclusion}}\text{.}$$

Ex 1: Rewrite the conditional statement in if-then form.

(a) All birds have feathers.

If an animal is a bird, then it has feathers.

(b) Two angles are supplementary if they are a linear pair.

If 2 angles are a linear pair, then they are supplementary.

(c) All whales are mammals.

If an animal is a whale, then it is a mammal.

(d) Three points are collinear if there is a line containing them.

If there is a line containing 3 points, then the points are collinear.

negation – the opposite of the original statement

*Conditional statements can be **true** or **false**.*

- ➔ To show a conditional statements is **TRUE**, you must prove that the conclusion is true every time the hypothesis is true.
- ➔ To show a conditional statement is **FALSE**, you need to give only one counterexample.

converse – the hypothesis and conclusion are *exchanged*

inverse – the hypothesis and conclusion are both *negated* in original conditional statement

contrapositive – the hypothesis and conclusion are both *negated* in the converse

Ex 2:

Conditional statement	If $m\angle A = 99°$, then $\angle A$ is obtuse.
Converse	If $\angle A$ is obtuse, then $m\angle A = 99°$.
Inverse	If $m\angle A \neq 99°$, then $\angle A$ is not obtuse.
Contrapositive	If $\angle A$ is not obtuse, then $m\angle A \neq 99°$.

Ex 3: Write the if-then form, the converse, the inverse, and the contrapositive of the statement: "Soccer players are athletes." Decide whether each statement is true or false.

Conditional statement	If you are a soccer player, then you are an athlete. $\boxed{T}$
Converse	If you are an athlete, then you are a soccer player. $\boxed{F}$
Inverse	If you are not a soccer player, then you are not an athlete. $\boxed{F}$
Contrapositive	If you are not an athlete, then you are not a soccer player. $\boxed{T}$

equivalent statements – two statements that are either **both** true or **both** false

- ➔ Conditional is always equivalent to Contrapositive
- ➔ Converse is always equivalent to Inverse

definition – a conditional statement whose original statement and its converse are **both** true

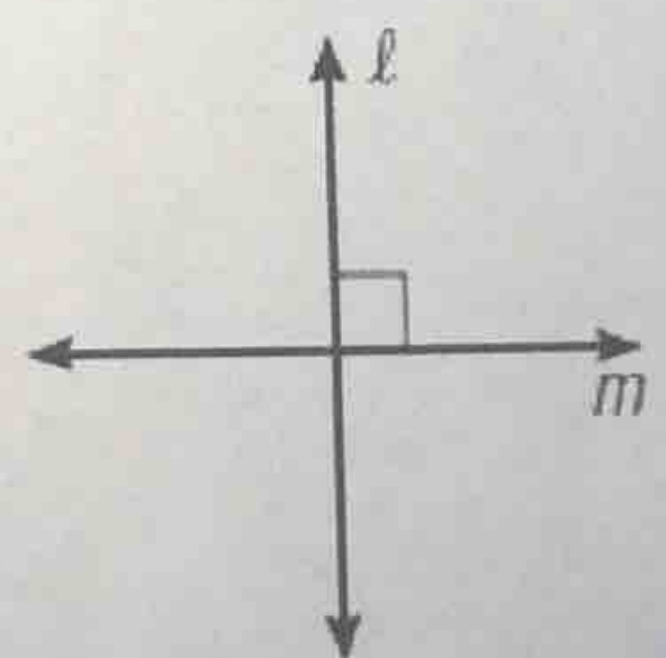
KEY CONCEPT — *For Your Notebook*

Perpendicular Lines

Definition If two lines intersect to form a right angle, then they are **perpendicular lines**.

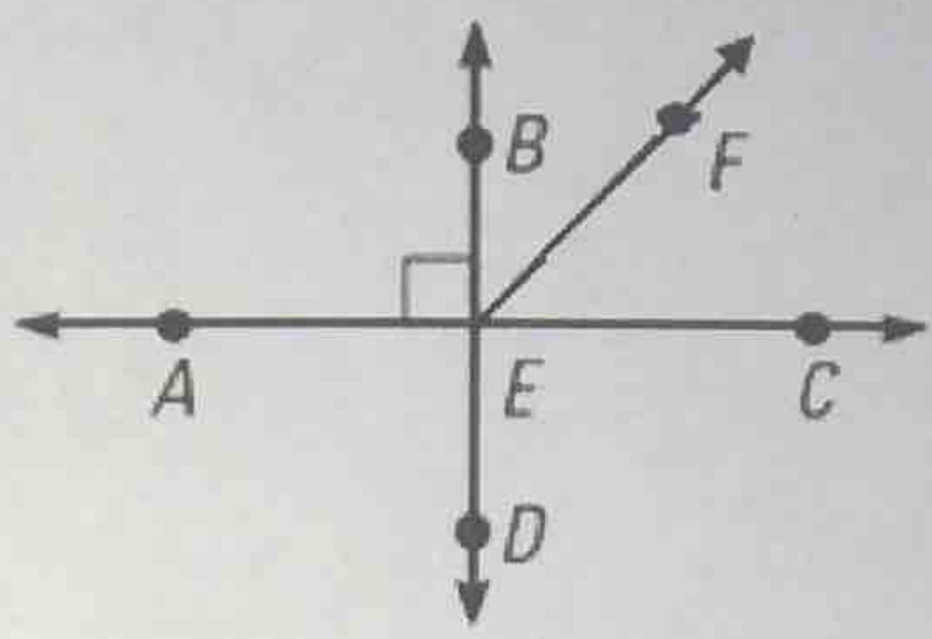
The definition can also be written using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.



$\ell \perp m$

Ex 4: Decide whether each statement about the diagram is true. Explain using the definitions you have learned.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$ TRUE. The right angle symbol indicates the lines form a right angle, so you can say they are perpendicular.

b. $\angle AEB$ and $\angle CEB$ are a linear pair. TRUE. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, so $\angle AEB$ and $\angle CEB$ are a linear pair.

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays. FALSE. Point E does not lie on the same line as A and B, so the rays are not opposite rays.

d. $\angle BEF$ and $\angle FEC$ are complementary. TRUE. By the angle addition postulate, $m\angle BEF + m\angle FEC = 90°$, so the angles are complementary by definition.

e. $\overrightarrow{FE}$ bisects $\angle CEB$ FALSE. The angles are not shown to be congruent so that does not satisfy the definition of an angle bisector.

biconditional statement – a statement that contains the phrase "if and only if" (because both the conditional statement and its converse are true); any valid definition can be written as a biconditional statement