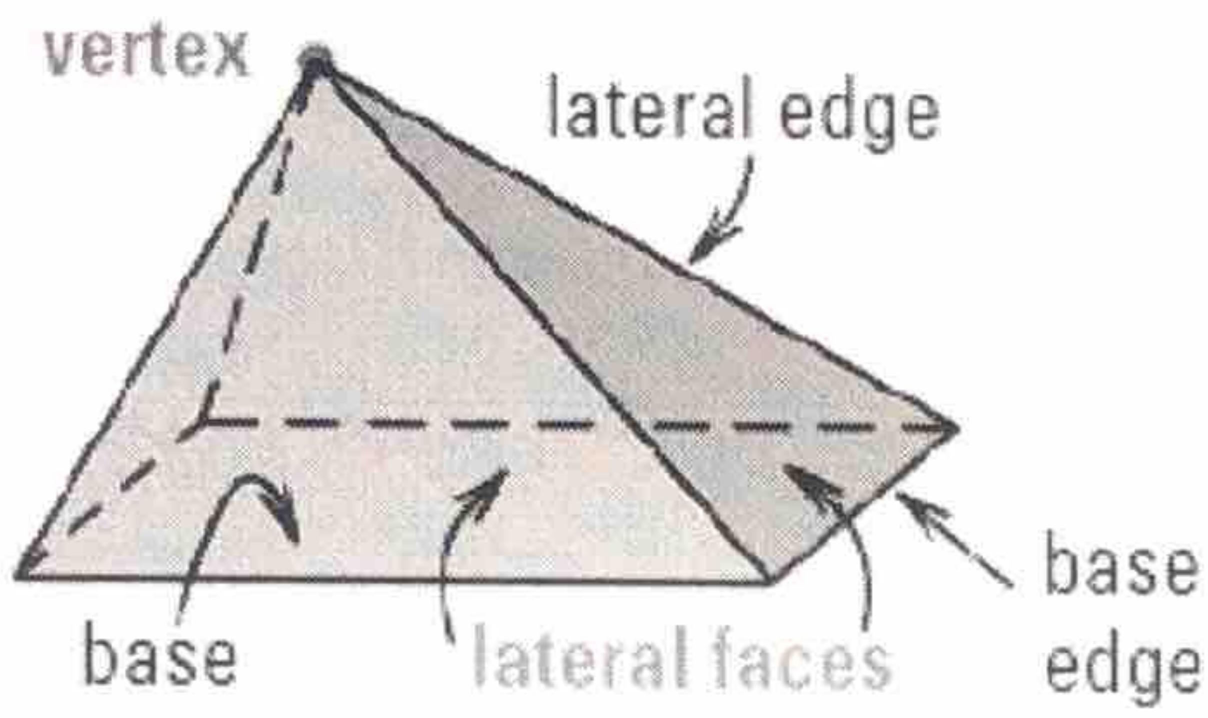
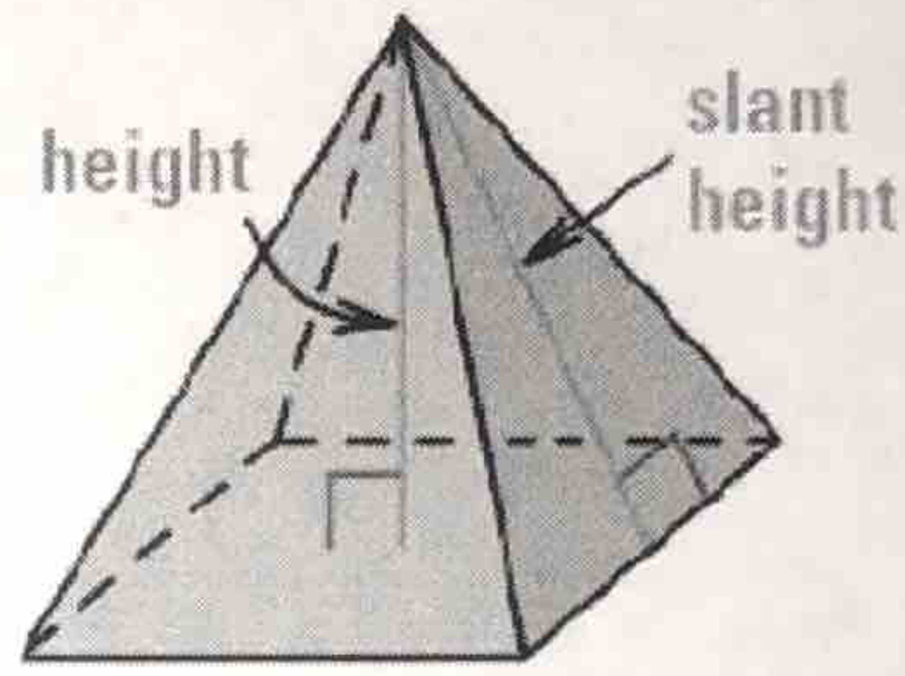
12.3 Surface Area of Pyramids and Cones

pyramid - a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex, called the **vertex of the pyramid**



Pyramid

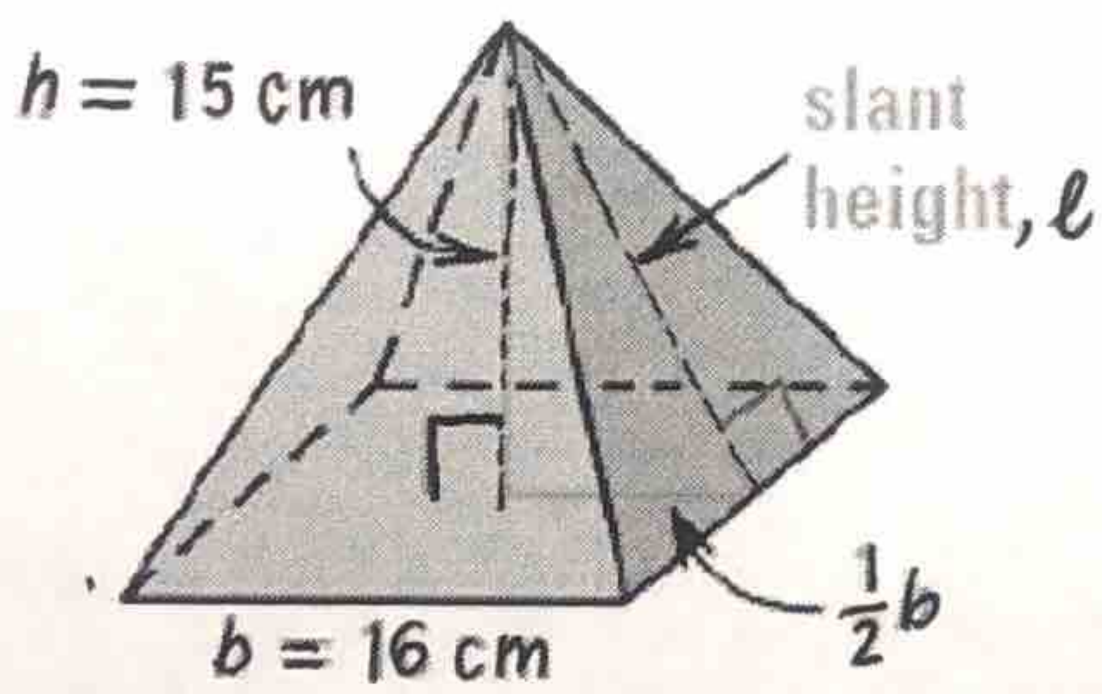


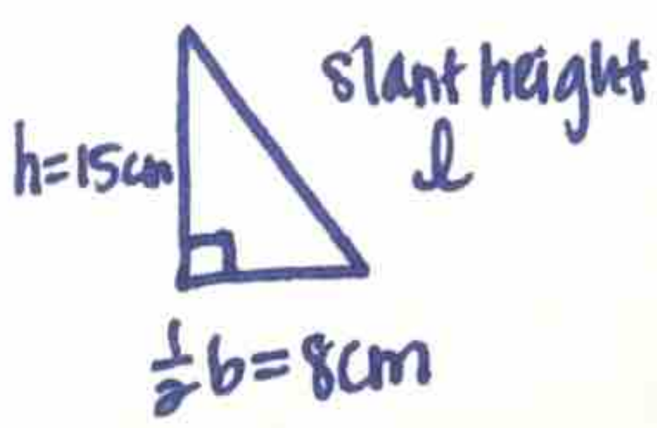
Regular pyramid

regular pyramid - has a regular polygon for a base, and the segment joining the vertex and the center of the base is perpendicular to the base; the lateral faces of a regular pyramid are congruent isosceles triangles

slant height - the height of a lateral face of a regular pyramid

Ex 1: Find the area of each lateral face of the pyramid.





$\ell^2 = h^2 + (\frac{1}{2}b)^2$

$\ell^2 = (15)^2 + (8)^2$

$\ell^2 = 289$

$\ell = 17$

The area of each triangular face is $A = \frac{1}{2}b\ell$

$\boxed{A = 136\text{cm}^2}$

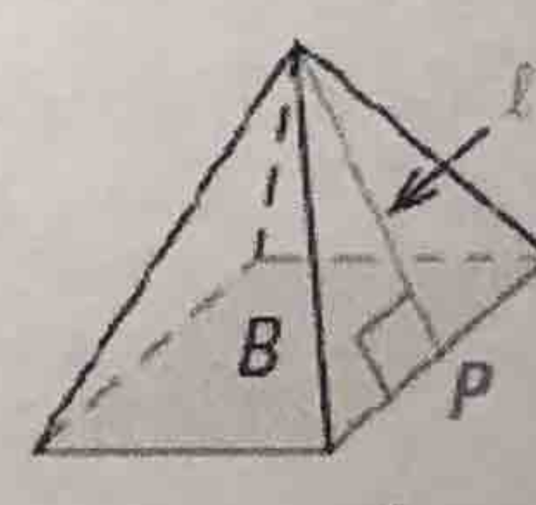
THEOREM *For Your Notebook*

THEOREM 12.4 Surface Area of a Regular Pyramid

The surface area S of a regular pyramid is

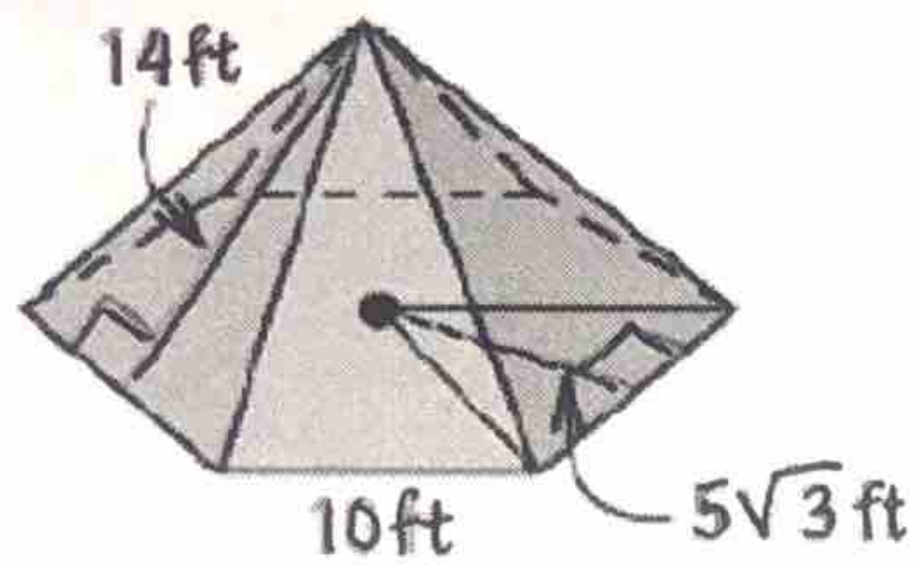
$$S = B + \frac{1}{2}P\ell,$$

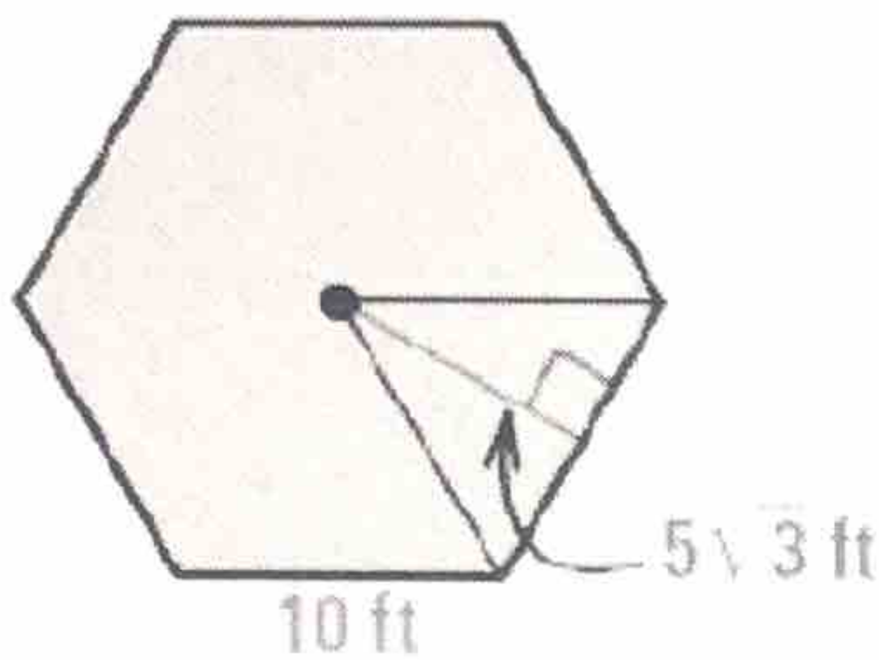
where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



$S = B + \frac{1}{2}P\ell$

Ex 2: Find the surface area of the regular hexagonal pyramid.





$A_{reg\,poly} = \frac{1}{2}aP$

$A_{\hexagon} = \frac{1}{2}(5\sqrt{3})(60)$

$A_{\hexagon} = 150\sqrt{3}\ ft^2 = B$

$SA = B + \frac{1}{2}P\ell$

$= 150\sqrt{3} + \frac{1}{2}(60)(14)$

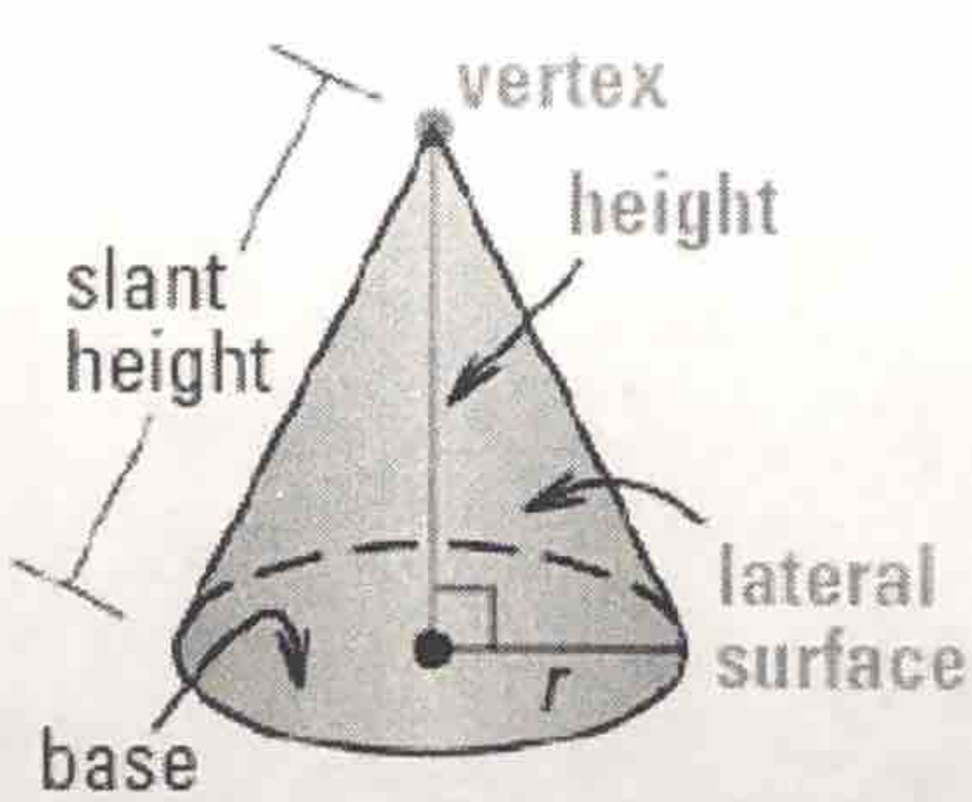
$= 150\sqrt{3} + 420$

$\approx \boxed{679.81\ ft^2}$

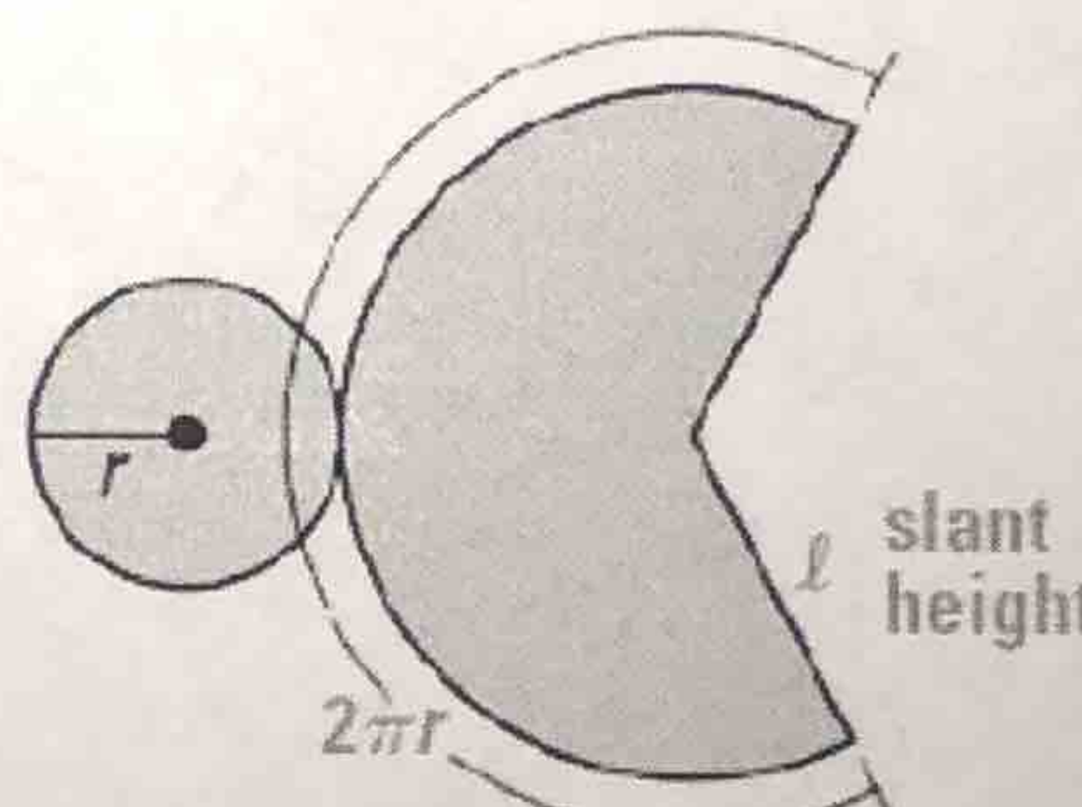
cone – has a circular base and vertex not in the same plane as the base

right cone – a cone in which the segment joining the vertex and the center of the base is perpendicular to the base, the slant height is the distance between the vertex and a point on the base edge

lateral surface of a cone – consists of all segments that connect the vertex with point on the base edge



Right cone



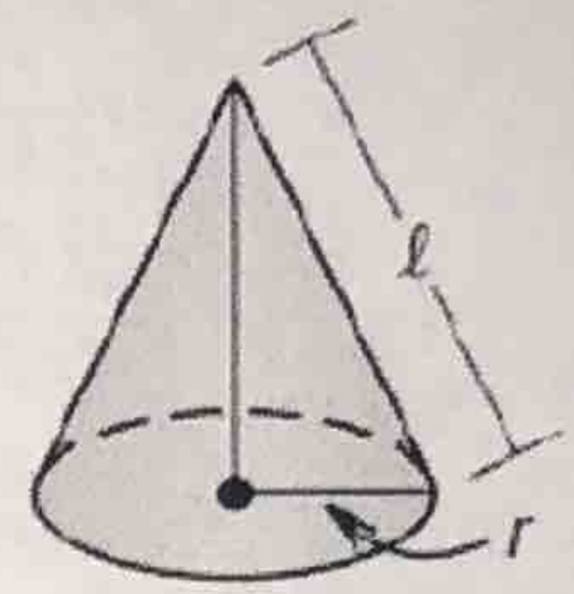
THEOREM *For Your Notebook*

THEOREM 12.5 Surface Area of a Right Cone

The surface area S of a right cone is

$$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell,$$

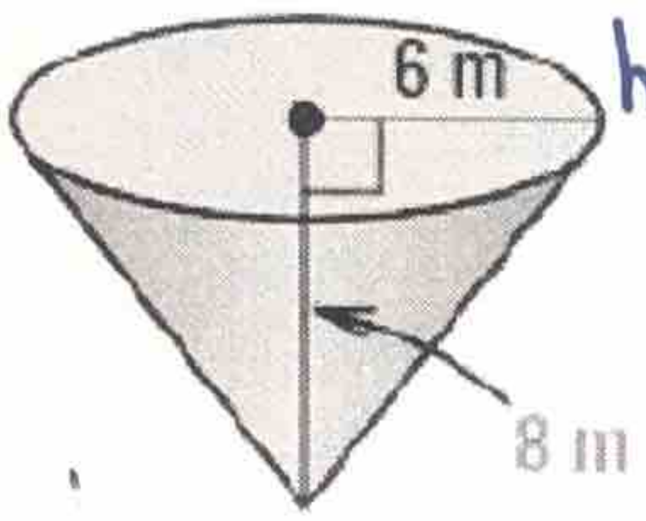
where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.

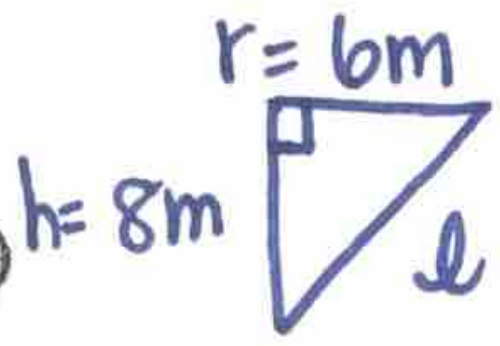


$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$

Lateral Area = $\pi r \ell$

Ex 3: What is the surface area of the right cone?





$\ell^2 = h^2 + r^2$
$\ell^2 = (8)^2 + (6)^2$
$\ell^2 = 100$
$\ell = 10$

$SA = \pi r^2 + \pi r \ell$
$SA = \pi(6)^2 + \pi(6)(10)$
$\boxed{SA = 96\pi \text{ m}^2}$
$\boxed{SA \approx 301.6 \text{ m}^2}$

Ex 4: The traffic cone can be approximated by a right cone with radius 5.7 inches and height 18 inches. Find the approximate lateral area of the cone.

$\ell^2 = (5.7)^2 + (18)^2$
$\ell^2 = 32.49 + 324$
$\ell^2 = 356.49$
$\ell \approx 18.88$ in

Lateral area = $\pi r \ell$
$= \pi(5.7)(18.9)$
$\approx \boxed{338 \text{ in}^2}$