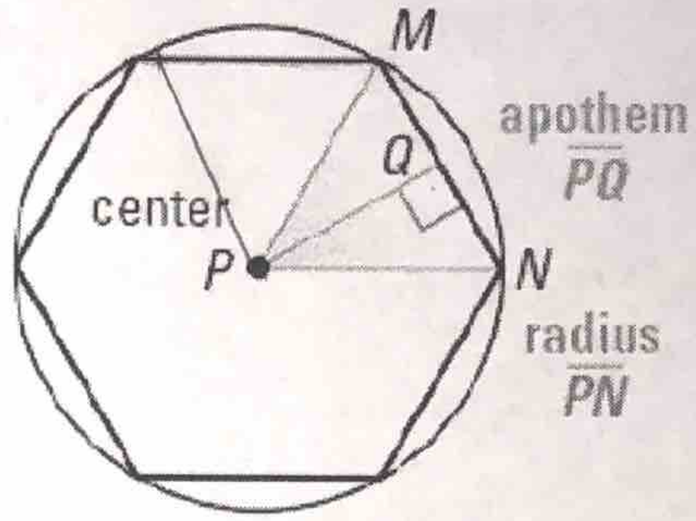
11.6 Areas of Regular Polygons

center of a polygon – center of a polygon's inscribed circle

radius of a polygon – radius of a polygon's inscribed circle

apothem of a polygon – the distance from the center to any side of the polygon



∠MPN is a central angle.

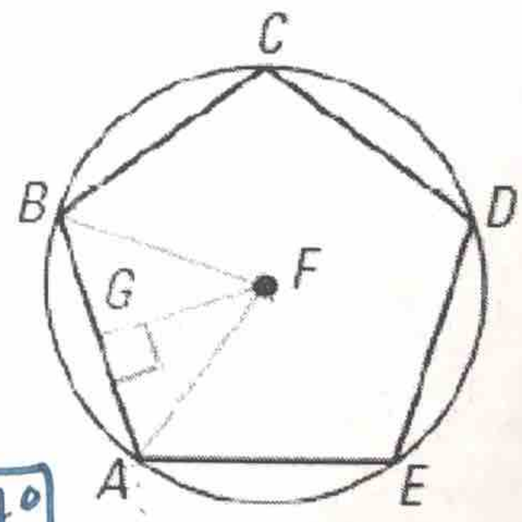
central angle of a regular polygon - angle formed by two radii drawn to consecutive vertices of the polygon

Ex 1: In the diagram, ABCDE is a regular pentagon inscribed in circle F. Find each angle measure.

(a) $m\angle AFB = \frac{360°}{5} = \boxed{72°}$

(b) $m\angle AFG = \frac{1}{2} m\angle AFB = \boxed{36°}$

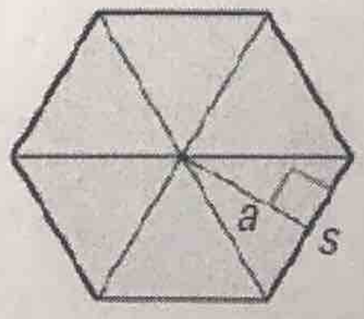
(c) $m\angle GAF = 180° - (90° + 36°) = \boxed{54°}$



THEOREM *For Your Notebook*

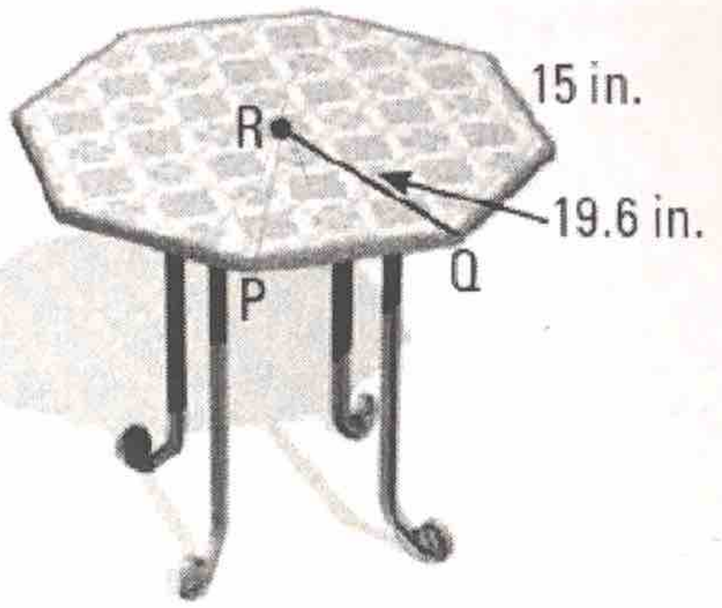
THEOREM 11.11 Area of a Regular Polygon

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$.



Ex 2: You are decorating the top of a table by covering it with small ceramic tiles. The tabletop is a regular octagon with 15-inch sides and a radius of about 19.6 inches. What is the area you are covering?

$P = 8(15) = 120$ in

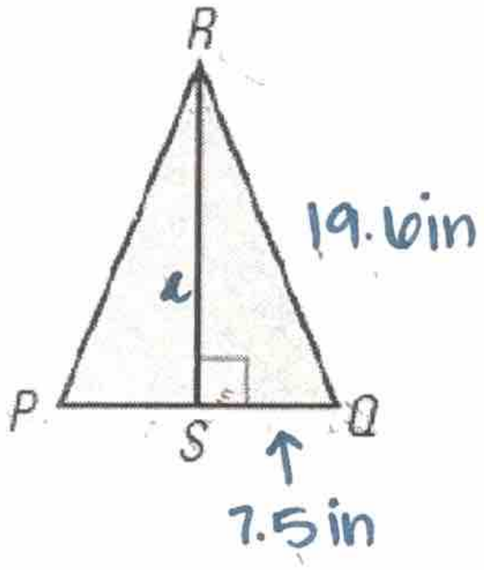


apothem: $a^2 + (7.5)^2 = (19.6)^2$

$a^2 + 56.25 = 384.16$

$a^2 = 327.91$

$a \approx 18.108$



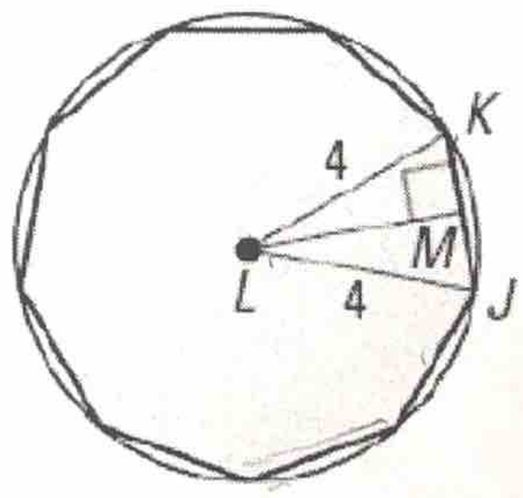
area: $A = \frac{1}{2} a P$

$= \frac{1}{2}(18.108)(120)$

$\boxed{A \approx 1086.5 \text{ in}^2}$

Ex 3: A regular nonagon is inscribed in a circle with radius 4 units. Find the perimeter and area of the nonagon.

$m\angle JLK = \frac{360^\circ}{9} = 40^\circ$



$\sin(20^\circ) = \frac{MK}{LK}$ $\quad \cos(20^\circ) = \frac{LM}{LK}$

$\sin(20^\circ) = \frac{MK}{4}$ $\quad \cos(20^\circ) = \frac{LM}{4}$

$MK \approx 1.368$ $\quad LM \approx 3.759$

$= \text{apothem}$

side length $= 2(1.368)$

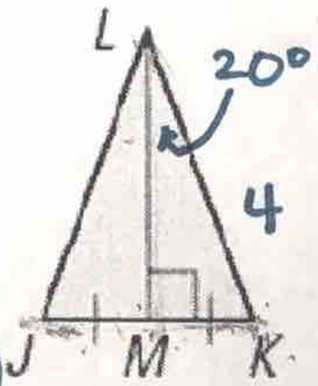
≈ 2.74

$A = \frac{1}{2} a P$

$A \approx \frac{1}{2}(3.759)(24.6)$

$P = 9(2.74)$

$\boxed{P \approx 24.6 \text{ units}}$ $\quad \boxed{A \approx 46.3 \text{ units}^2}$



Methods for solving right triangles:

- Pythagorean Theorem,
- Special Right Triangles: 45°-45°-90°, 30°-60°-90°,
- or Trigonometry