11.6 Areas of Regular Polygons

center of a polygon - center of a polygon's inscribed circle

radius of a polygon - radius of a polygon's inscribed circle

apothem of a polygon - the distance from the center to any side of the polygon

central angle of a regular polygon - angle formed by two radii drawn to consecutive vertices of the polygon

Ex. 1: In the diagram, ABCDE is a regular pentagon inscribed in circle F. Find each angle measure.

(a) \( \angle AFB = \frac{360\degree}{5} = 72\degree \)

(b) \( \angle AFG = \frac{1}{2} \angle AFB = 36\degree \)

(c) \( \angle GAF = 180\degree - (90\degree + 36\degree) = 54\degree \)

THEOREM

THEOREM 11.11 Area of a Regular Polygon

The area of a regular \( n \)-gon with side length \( s \) is half the product of the apothem \( a \) and the perimeter \( P \), so \( A = \frac{1}{2} aP \), or \( A = \frac{1}{2} a \cdot ns \).
Ex 2: You are decorating the top of a table by covering it with small ceramic tiles. The tabletop is a regular octagon with 15-inch sides and a radius of about 19.6 inches. What is the area you are covering?

\[ P = 8(15) = 120 \text{ in} \]

**Apothem:**

\[ a^2 + (7.5)^2 = (19.6)^2 \]
\[ a^2 + 56.25 = 384.16 \]
\[ a^2 = 327.91 \]
\[ a \approx 18.108 \]

**Area:**

\[ A = \frac{1}{2} aP \]
\[ = \frac{1}{2}(18.108)(120) \]
\[ A \approx 1086.5 \text{ in}^2 \]

Ex 3: A regular nonagon is inscribed in a circle with radius 4 units. Find the perimeter and area of the nonagon.

\[ m\angle JLP = \frac{360^\circ}{9} = 40^\circ \]

\[ \sin(20^\circ) = \frac{MK}{LK} \]
\[ \sin(20^\circ) = \frac{MK}{4} \]

\[ MK \approx 1.368 \]

**Side length:**

\[ \approx 2.74 \]

\[ P = 9(2.74) \]
\[ P \approx 24.6 \text{ units} \]

Methods for solving right triangles:

- Pythagorean Theorem,
- Special Right Triangles: 45°-45°-90°, 30°-60°-90°,
- or Trigonometry