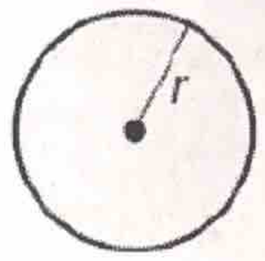
11.5 Areas of Circles and Sectors

THEOREM — *For Your Notebook*

THEOREM 11.9 Area of a Circle

The area of a circle is π times the square of the radius.



$A = \pi r^2$

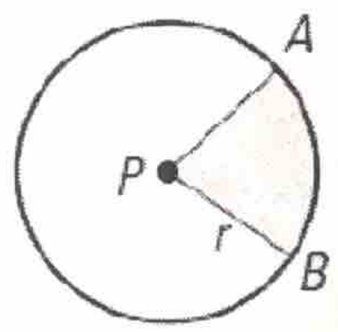
Justification: Ex. 43, p. 761; Ex. 3, p. 769

sector of a circle - the region bounded by two radii of the circle and their incepted arc

THEOREM — *For Your Notebook*

THEOREM 11.10 Area of a Sector

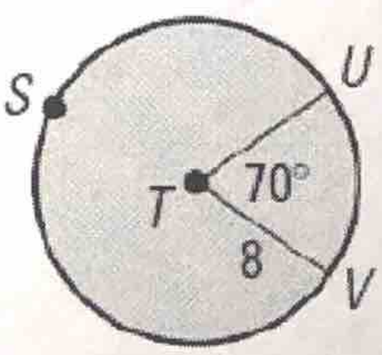
The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.



$$\text{Area of sector } APB = \frac{mAB}{360^\circ}, \text{ or Area of sector } APB = \frac{mAB}{360^\circ} \cdot \pi r^2$$
$$\pi r^2$$

Ex 1: Find the areas of the sectors formed by ∠UTV.

$m\angle UTV = 70^\circ$ so $m\overset{\frown}{USV} = 360^\circ - 70^\circ = 290^\circ$

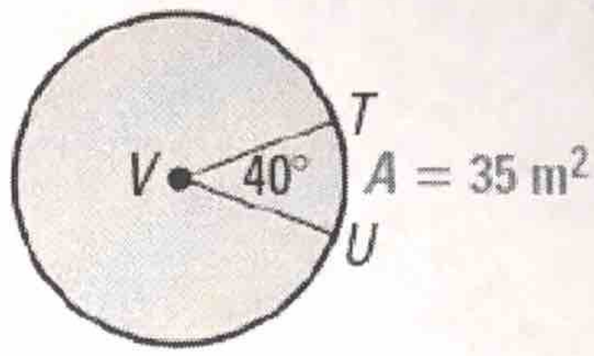


$A_{small} = \frac{m\overset{\frown}{UV}}{360^\circ}(\pi r^2)$ $A_{large} = \frac{m\overset{\frown}{USV}}{360^\circ}(\pi r^2)$

$= \frac{70^\circ}{360^\circ}\cdot[\pi(8)^2]$ $= \frac{290^\circ}{360^\circ}\cdot(\pi(8)^2)$

$\approx \boxed{39.10 \text{ units}^2}$ $\approx \boxed{161.97 \text{ units}^2}$

Ex 2: Find the area of circle V.



$$\text{area of sector} = \frac{m\widehat{TU}}{360^\circ}(\pi r^2)$$

$$35 = \frac{40^\circ}{360^\circ}(\pi r^2)$$

$$r^2 = \frac{315}{\pi}$$

$$r \approx 10$$

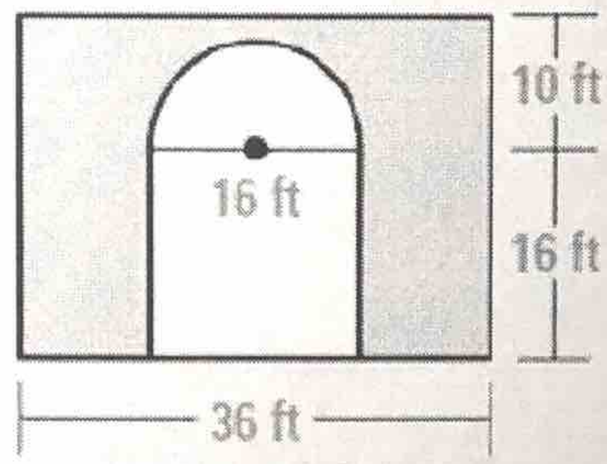
* πr^2 = area of the circle

$$A_\odot = \pi r^2$$

$$= \pi(10)^2$$

$$= \boxed{314\ m^2}$$

Ex 3: A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?



$A_{\square} = (16)(16)$

$= 256$

$A_{\square} = (36)(26)$

$= 936$

$A_\odot = \pi(8)^2$

$= 64\pi \Rightarrow A_{\frac{1}{2}\odot} = 32\pi$

$A_{\cap} = 936 - (256 + 32\pi)$

$\boxed{A \approx 579\ ft^2}$