11.5 Areas of Circles and Sectors

**THEOREM**

**THEOREM 11.9 Area of a Circle**

The area of a circle is \( \pi \) times the square of the radius.

\[
A = \pi r^2
\]

*Justification:* Ex. 43, p. 761; Ex. 3, p. 769

**sector of a circle** - the region bounded by two radii of the circle and their incepted arc

**THEOREM**

**THEOREM 11.10 Area of a Sector**

The ratio of the area of a sector of a circle to the area of the whole circle \( \pi r^2 \) is equal to the ratio of the measure of the intercepted arc to 360°.

\[
\text{Area of sector } APB = \frac{m_{\text{AB}}}{360^\circ} \cdot \pi r^2
\]

**Ex.1:** Find the areas of the sectors formed by \( \angle UTV \).

\[
m \angle UTV = 70^\circ \quad \text{so} \quad m \angle USV = 360^\circ - 70^\circ = 290^\circ
\]

\[
A_{\text{small}} = \frac{m_{\text{UV}}}{360^\circ} \cdot (\pi r^2)
\]

\[
= \frac{70^\circ}{360^\circ} \cdot \pi (8^2)
\]

\[
\approx 39.10 \text{ units}^2
\]

\[
A_{\text{large}} = \frac{m_{\text{USV}}}{360^\circ} \cdot (\pi r^2)
\]

\[
= \frac{290^\circ}{360^\circ} \cdot \pi (8^2)
\]

\[
\approx 161.97 \text{ units}^2
\]
Ex 2: Find the area of circle V.

\[
\text{area of sector} = \frac{m \hat{T}U}{360^\circ} (\pi r^2)
\]

\[
35 = \frac{40^\circ}{360^\circ} (\pi r^2)
\]

\[
r^2 = \frac{315}{\pi}
\]

\[
r \approx 10
\]

\[
A_0 = \pi r^2
\]

\[
= \pi (10)^2
\]

\[
= 314 \text{ m}^2
\]

* \(\pi r^2 = \text{area of the circle}\)

Ex 3: A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?

\[
A_\square = (16)(16) = 256
\]

\[
A_\square = (36)(26) = 936
\]

\[
A_0 = \pi (8)^2 = 64\pi \Rightarrow A_{\text{ent}} = 32\pi
\]

\[
A = 936 - (256 + 32\pi)
\]

\[
A \approx 579 \text{ ft}^2
\]