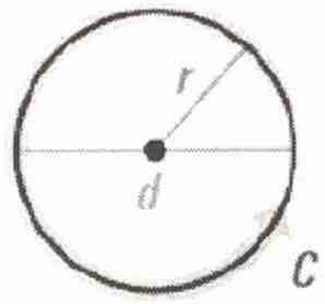
11.4 Circumference and Arc Length

circumference - the distance around a circle, for all circles ratio of circumference to diameter is the same - π!

THEOREM *For Your Notebook*

THEOREM 11.8 Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



Justification: Ex. 2, p. 769

$C = \pi d = 2\pi r$

Ex 1: The dimensions of a car tire are shown. To the nearest foot, how far does the tire travel when it makes 15 revolutions?

diameter $= 15 + 2(5.5) = 26$ in

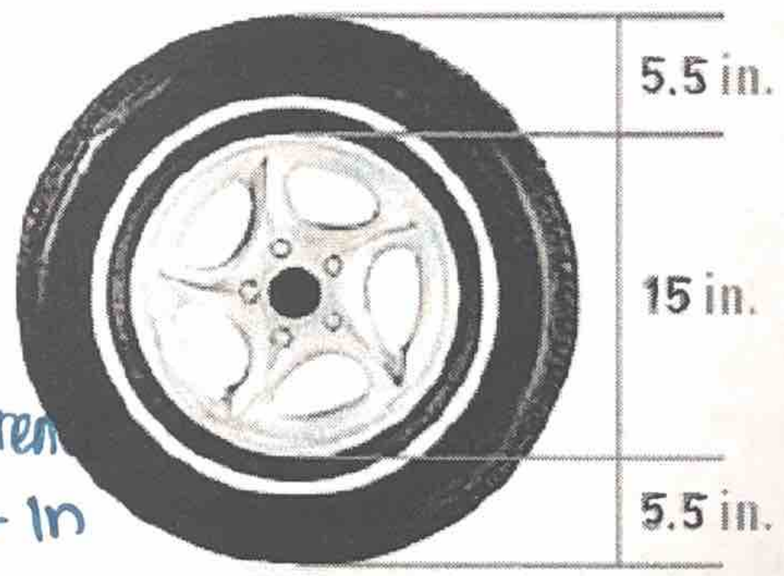
circumference $= \pi d = \pi(26)$

≈ 81.68 in

distance = # of revolutions · Circumfere

$\approx 15(81.68) \approx 1225.2$ in

$1225.2 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 102.1 \text{ ft} \approx$ **102 ft**

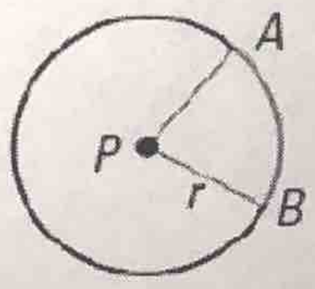


arc length - a portion of the circumference of a circle; arcs are measured in degrees, arc lengths are measured in linear units

COROLLARY *For Your Notebook*

ARC LENGTH COROLLARY

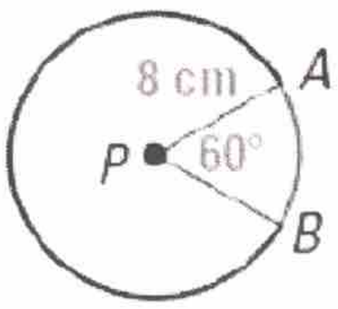
In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.



$$\frac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360^\circ}, \text{ or Arc length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot 2\pi r$$

Ex 2: Find the length of each arc.

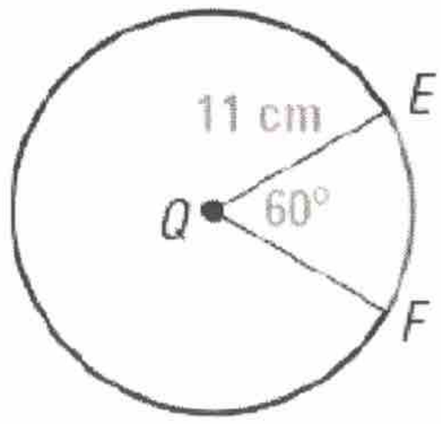
a.



arc length $= \frac{60°}{360°}\,[2\pi(8)]$

$\approx$ **8.38 cm**

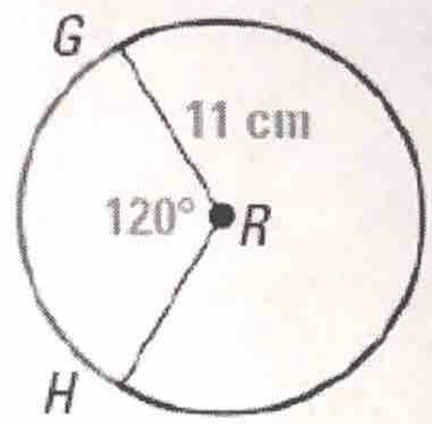
b.



arc length $= \frac{60°}{360°}\,[2\pi(11)]$

$\approx$ **11.52 cm**

c.

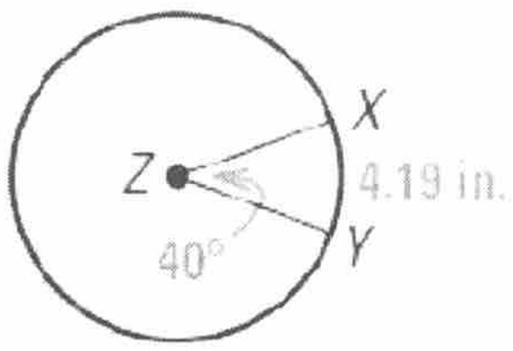


arc length $= \frac{120°}{360°}\,[2\pi(11)]$

$\approx$ **23.04 cm**

Ex 3: Find the indicated measure.

a. Circumference C of $\odot Z$

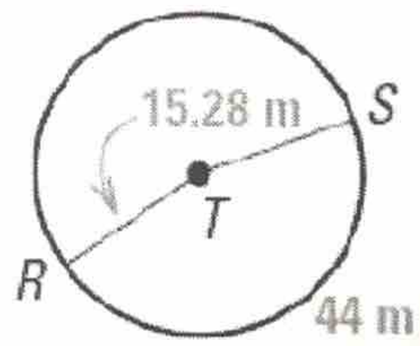


$\frac{40°}{360°} = \frac{1}{9}$ of circle

so, $4.19 = \frac{1}{9}C$

$C = 37.71$ in

b. $m\widehat{RS}$



$\frac{m\widehat{RS}}{360°}\,[2\pi(15.28)] = 44$

$m\widehat{RS} \approx 165°$

* when the equation of a circle is given, its standard form is:

$(x - h)^2 + (y - k)^2 = r^2$, where

(h, k) is the center of the circle and r is the radius