11.3 Perimeter and Area of Similar Figures

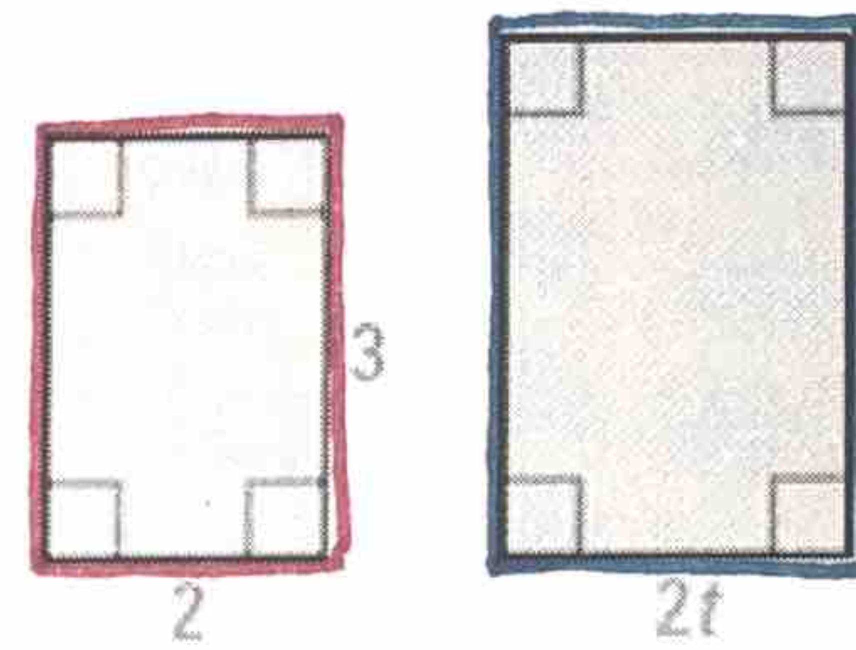
Recall: If 2 polygons are similar, then the ratio of their perimeters, or of any 2 corresponding lengths, is equal to the ratio of their corresponding side lengths.

Ratio of perimeters

$$\frac{\text{Blue}}{\text{Red}} = \frac{10t}{10} = t$$

Ratio of areas

$$\frac{\text{Blue}}{\text{Red}} = \frac{6t^2}{6} = t^2$$



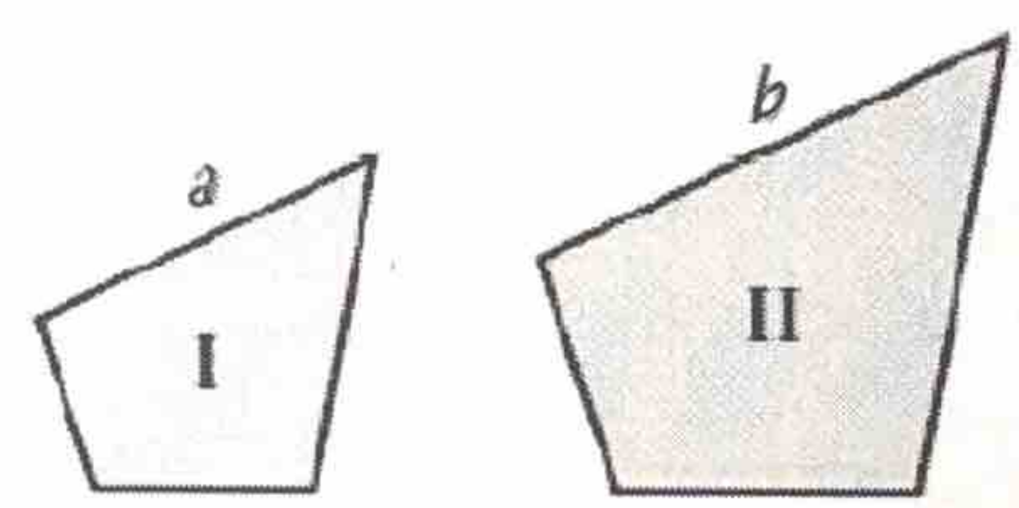
THEOREM *For Your Notebook*

THEOREM 11.7 Areas of Similar Polygons

If two polygons are similar with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$.

$$\frac{\text{Side length of Polygon I}}{\text{Side length of Polygon II}} = \frac{a}{b}$$

$$\frac{\text{Area of Polygon I}}{\text{Area of Polygon II}} = \frac{a^2}{b^2}$$

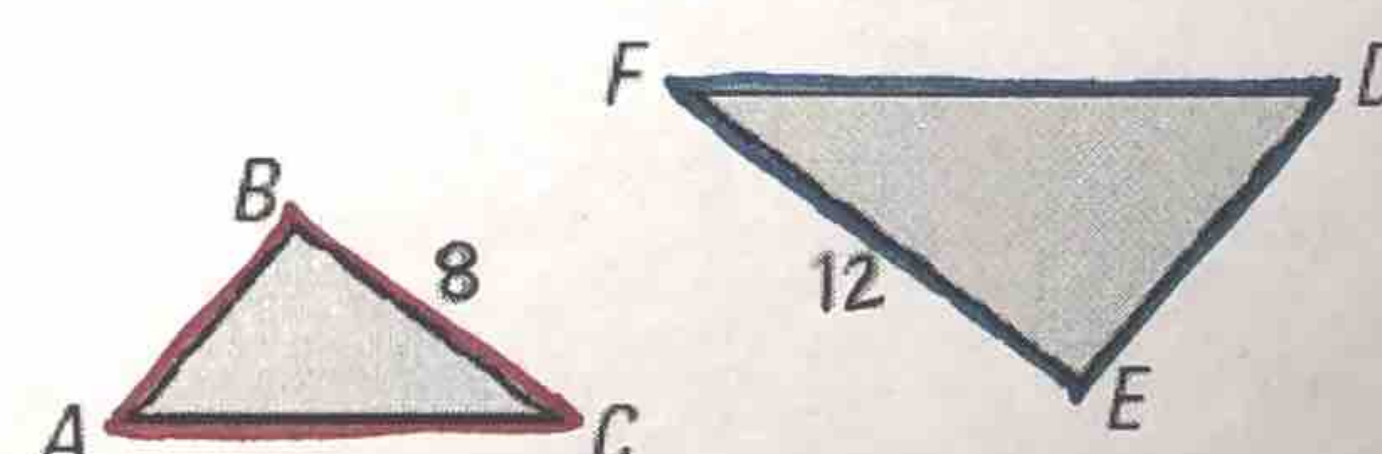


Polygon I ~ Polygon II

Justification: Ex. 30, p. 742

Ex 1: In the diagram, $\triangle ABC \sim \triangle DEF$. Find the indicated ratio.

(a) Ratio (red to blue) of the perimeters



By Theorem 6.1, ratio of sides = ratio of perimeters

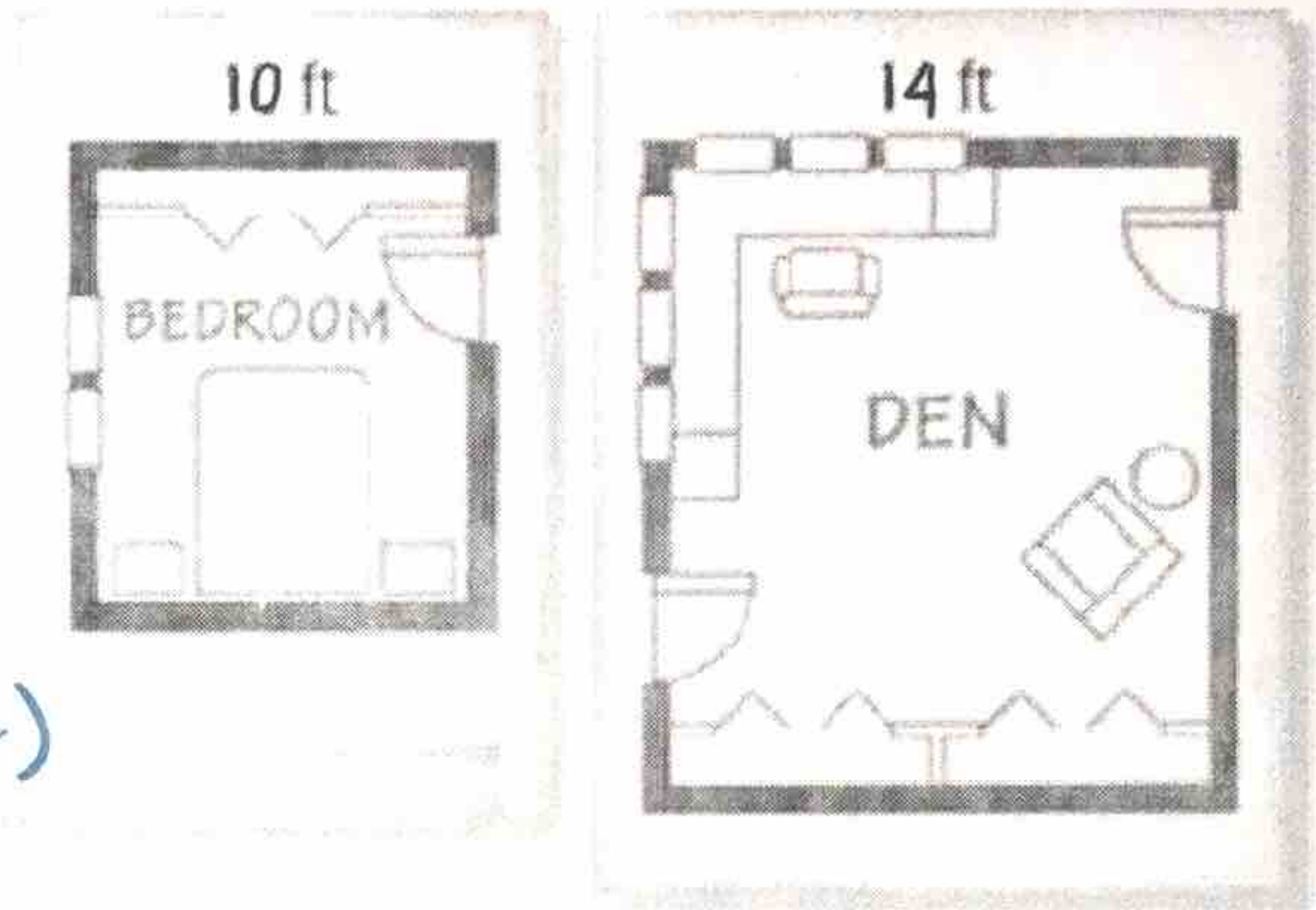
$\frac{8}{12} = \boxed{\frac{2}{3}}$

(b) Ratio (red to blue) of the areas

By Theorem 11.7, ratio of areas $= \frac{a^2}{b^2}$ when ratio of sides is $\frac{a}{b}$

$\frac{2}{3} \Rightarrow \frac{2^2}{3^2} = \boxed{\frac{4}{9}}$

Ex 2: You are installing the same carpet in a bedroom and a den. The floors of the rooms are similar. The carpet for the bedroom costs $225. Carpet is sold by the square foot. How much does it cost to carpet the den?



ratio of sides $= \frac{14}{10} = \frac{7}{5}$

so ratio of areas $= \frac{7^2}{5^2} = \frac{49}{25}$

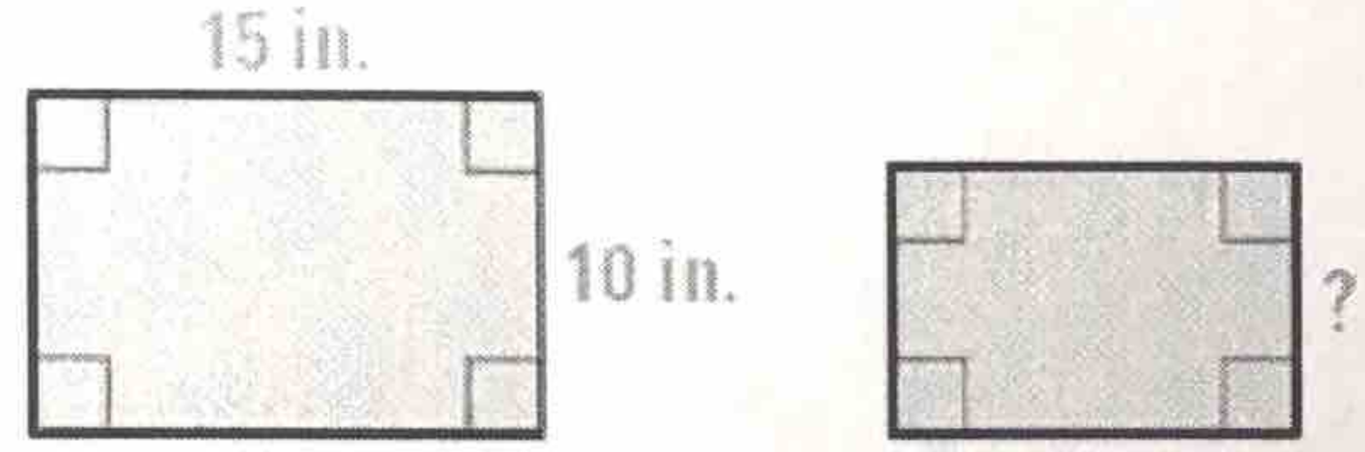
$$\frac{\text{den (area)}}{\text{bedroom (area)}} = \frac{\text{den (cost)}}{\text{bedroom (cost)}}$$

$$\frac{49}{25} = \frac{x}{225}$$

$$25x = 11025$$

$$x = \boxed{\$441}$$

Ex 3: A large rectangular baking pan is 15 inches long and 10 inches wide. A smaller pan is similar to the large pan. The area of the smaller pan is 96 square inches. Find the width of the smaller pan.



$A = (15)(10) = 150\text{ in}^2$ $\quad$ $A = 96\text{ in}^2$

$$\frac{\text{area of small}}{\text{area of large}} = \frac{96}{150} = \frac{16}{25}$$

so $\frac{\text{width of small}}{\text{width of large}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

ratios of widths: $\frac{4}{5} = \frac{x}{10}$

$$5x = 40$$

$$\boxed{x = 8\text{ in}}$$

* All regular polygons with the same number of sides are similar.
* All circles are also similar.

Ex 4: The floor of the gazebo show is a regular octagon. Each side of the floor is 8 feet, and the area is about 309 square feet. You build a small model gazebo in the shape of a regular octagon. The perimeter of the floor of the model gazebo is 24 inches. Find the area of the floor of the model gazebo to the nearest tenth of a square inch.

all regular octagons are similar

ratio of sides = ratio of perimeters

$$= \frac{8(8\,ft)}{24\,in}$$

$$= \frac{64\,ft}{24\,in}$$

$$= \frac{64\,ft}{2\,ft}$$

$$= \frac{32}{1}$$

ratio of areas $= \frac{32^2}{1^2}$

$$= \frac{1024}{1}$$

calculate model gazebo area: $\frac{1024}{1} = \frac{309\,ft^2}{x\,ft^2}$

$$1024x = 309$$

$$x \approx 0.302\,ft^2$$

convert to inches: $0.302\,\cancel{ft^2} \cdot \frac{144\,in^2}{1\,\cancel{ft^2}} \approx \boxed{43.5\ in^2}$